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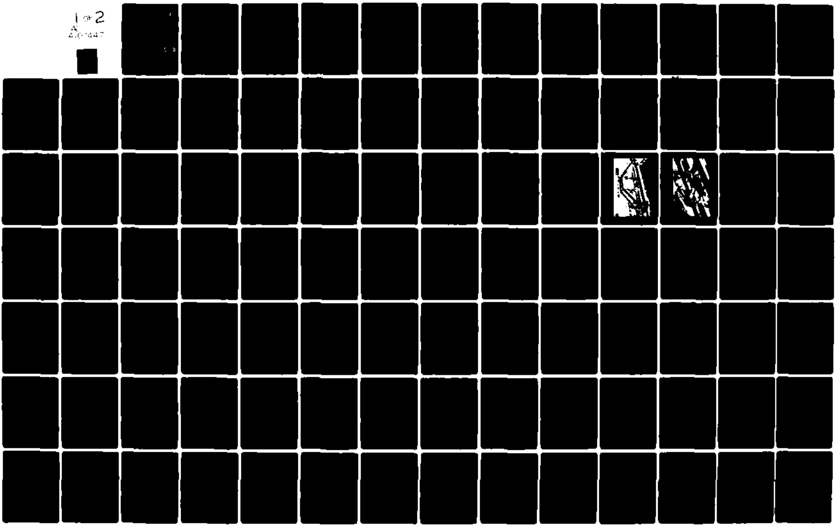

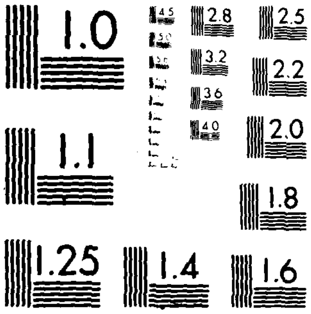

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IDENTIFICATION OF WHEELSET/RAIL CREEP COEFFICIENTS FROM DYNAMIC RESPONSE DATA USING THE MAXIMUM LIKELIHOOD PARAMETER IDENTIFICATION TECHNIQUE

by

William N. Herzog
Princeton University
School of Engineering and Applied Science
Department of Mechanical and Aerospace Engineering

169

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Engineering from Princeton University, 1978.

Prepared by:

William N Herzog
Willian N. Herzog

Approved by:

Larry M Sweet
Professor Larry M. Sweet

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Air Force Institute of Technology (ATC)
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ABSTRACT

This thesis explores the application of the maximum likelihood parameter identification technique to determine the wheel/rail creep coefficients using dynamic response data. The equations of motion for the dynamically scaled wheelset are presented and the reduced form of the maximum likelihood equations as applicable to the dynamically scaled wheelset model are developed. The maximum likelihood equations were formulated into a maximum likelihood algorithm which was implemented in Fortran IV. Using simulated wheelset data, the effects of a random input representation of the track versus a deterministic input with uncertainty representation are determined. The effects of various levels of measurement noise are also examined. This preliminary analysis indicates that the deterministic representation of the track input yields better results. Representing the track as a random track input requires further investigation into the effects of longer data records and smaller time steps on the performance of the maximum likelihood algorithm.

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This thesis carries 1408-T in the records of the Department of Mechanical and Aerospace Engineering.

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Chapter 2

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NOMENCLATURE

A	a general matrix
B	covariance matrix of ν
$E[f]$	expected value of f
F	system dynamics matrix for dynamically scaled wheelset model
G	system input matrix for dynamically scaled wheelset model
H	measurement matrix for dynamically scaled wheelset model
K	Kalman gain matrix
K_y, K_ψ	lateral (yaw) spring constants, n/m (n-m/rad)
L	likelihood function
M	Fisher information matrix
M	mass of dynamically scaled wheelset model, kg
N	end of summation index for likelihood function
$N(x,y)$	Gaussian distribution with mean x and covariance matrix y
P	error covariance of Kalman Filter state estimates
Q	covariance matrix of random input disturbance
R	covariance matrix of measurement noise
T	period of a signal
V	translational velocity of the dynamically scaled wheelset model
$\underline{Y}$	transformation used to solve matrix Riccati equation and $\partial P/\partial \underline{\theta}$ using a Kalman-Engler solution technique
a	element of a general matrix
b	bias in the yaw potentiometer mounted on the dynamically scaled wheelset model

c	lateral damping coefficient, n-sec/m
e	electrical bias added to the yaw potentiometer output by the on-board analog computer
f	creep coefficient, n
i	summation index, product index
j	imaginary part of a complex number
ℓ	distance from wheelset c.g. to contact point between wheel and rail, m
m	slope of the straight line function which describes the characteristics of the displacement transducers
p	probability density
r_o	wheel rolling radius, m
t	time
$\underline{u}$	deterministic input into a dynamic system
$\underline{v}$	measurement noise vector
$\underline{w}$	random input into a dynamic system
$\underline{x}$	vector of state variables for a dynamic system
$\underline{\hat{x}}$	vector of estimates of the state variables of a dynamic system
y	lateral displacement of wheelset, m
α	wheel conicity, rad
β	factor which multiplies the nominal values for the creep coefficients to get the true values for the creep coefficients
Γ	matrix which multiplies the deterministic input in the Kalman Filter equation for the propagation of the state variable estimates
$\underline{\gamma}$	transformation used to derive $\partial P/\partial \underline{\theta}$ using the Kalman-Engler solution technique
Δ	small increment
δ	distance from rail to inertial reference

δ_1, δ_2	distance of left and right rails from fixed reference, m
$\bar{\delta}$	distance of track centerline from reference (= $(\delta_1 + \delta_2)/2$), m
ε	error criterion
ξ	term of the likelihood function
η	yaw potentiometer signal after processing on the on-board analog computer
$\underline{\theta}$	vector of unknowns to be identified using the maximum likelihood parameter identification technique
Θ	matrix which multiplies the measurement vector in the Kalman Filter equation for the propagation of the state variable estimates
Λ	matrix which multiplies the state variable estimates in the equation for the propagation of the sensitivity vector $\partial\hat{\underline{x}}/\partial\underline{\theta}$
Π	product
Σ	summation
T	matrix which multiplies the measurement vector in the equation for the propagation of the sensitivity vector $\partial\hat{\underline{x}}/\partial\underline{\theta}$
Φ	state transition matrix
Ψ	matrix which multiplies the deterministic input in the equation for the propagation of the sensitivity vector $\partial\hat{\underline{x}}/\partial\underline{\theta}$
Ω	axle rotational velocity (= V/r_o), sec^{-1}
$\underline{\lambda}$	transformation used in solving the matrix Riccati equation using the Kalman-Engler solution technique
ν	the innovation $\underline{z} - H\hat{\underline{x}}$
σ	eigenvalues of the dynamically scaled wheelset model system dynamics matrix

ϕ	element of Φ
ψ	yaw angle of dynamically scaled wheelset model

Subscripts

i	discrete point in time
L	left side
n	last of a sequence of discrete value
r	right side
y	lateral
ss	steady state
ψ	yaw
o	starting point
11	subscript of longitudinal creep coefficient
22	subscript of lateral creep coefficient

Sub-subscripts

o	nominal value

Superscript

*	true value

INTRODUCTION

The dynamic response of railroad vehicles to track irregularity inputs is determined by interactions between the vehicle body and suspension components, kinematic constraints dependent on the profiles of the wheels and rails, and friction forces generated at the wheel/rail interface. These latter forces, known as creep forces, result from small differential velocities between wheel and rail in directions tangential and lateral to the wheel. Linearized dynamic models for rail vehicles represent creep forces using the slope of the creep force versus relative velocity function at the origin; the slopes are known in the literature as creep coefficients.

The values of these coefficients has been determined from Hertzian contact theory by Kalker (4) and by measurements under steady-state conditions (9). However, previous analysis (12) of *dynamic* response data has indicated that better agreement between theory and experiment is obtained if values for creep coefficients are used that are up to 50% lower than determined in (4) and (9). The "best" values for creep coefficients under dynamic conditions have not been determined with precision; such determination would be of great value to the rail research technical community.

The objective of this thesis is the estimation of creep coefficients from experimental vehicle response data using advanced parameter identification techniques. To focus the effort on estimation of creep coefficients, data used are measurements of the response of a simplified scale model vehicle, whose characteristics and parameters are well-understood, exclusive of the creep coefficients. Use of the simplified model, in this case a wheelset (single axle with two wheels and lateral and yaw suspension, capable of two degrees-of-freedom motion), permits examination of the fundamental kinematic hunting and instability characteristic of rail vehicles, without the effects of more complex vehicle configurations obscuring the results. The use of dynamically scaled models (9) allows better control of experimental conditions than expected in full scale.

The method used to estimate the creep coefficients is the maximum likelihood parameter identification technique. This technique has been used to identify aircraft stability derivatives successfully for several years, and represents the state-of-the-art in parameter estimation. The specific objective of this thesis is to develop and implement the maximum likelihood method for dynamically scaled wheelset models. In addition some insight into the best way to conduct dynamic experiments involving the dynamically scaled wheelset model was desired.

This research program was divided into two phases. The first was the experimental phase during which the dynamic wheelset experiments were conducted. The purpose of the experiments was to obtain data that could be processed using a maximum likelihood processor. The second stage of the research project was a developmental one. During this part of the research program the maximum likelihood algorithm was developed and implemented into a Fortran IV computer program. The final phase of research was the test phase during which simulated wheelset data was generated using a computer model of the dynamically scaled wheelset model. The maximum likelihood computer program was then checked out using this simulated data.

The experimental phase was conducted first so that as much information as possible about the wheelset model could be obtained prior to the development of the maximum likelihood program. The actual wheelset data taken was not analyzed using the maximum likelihood program. The results for the simulated data suggest that more research needs to be done with simulated data so that more conclusive results can be drawn when the actual wheelset data is processed.

The body of this thesis is divided into three chapters. The first presents the generalized maximum likelihood equations and then develops the reduced equations that apply to the dynamically scaled wheelset model. The second chapter discusses the experimental phase of the research program. The knowledge gained from these initial experiments will be used to develop an improved experimental program for the next series of data. The final chapter presents the results obtained for the simulated data, and the interpretation of these results in terms of actual wheelset data.

Chapter 1

MAXIMUM LIKELIHOOD THEORY

1.1 General Form of the Maximum Likelihood Equations

The maximum likelihood parameter identification technique is used to identify unknown parameters of a dynamic system given measurements of some or all of the system's state variables. This method maximizes the probability of the given measurements conditioned on the vector of unknowns. The following development of the maximum likelihood equations will be for a general case, linear, time-varying system. Except where otherwise noted the equations in this section were taken from References (8) and (3).

The state-space form of the differential equations for a linear dynamic system is

$$\dot{\underline{x}}(t) = F(t)\underline{x}(t) + L(t)\underline{u}(t) + G(t)\underline{w}(t) \tag{1.1}$$

where $\underline{x}(t)$ = state variable vector

$\underline{u}(t)$ = known input vector

$\underline{w}(t)$ = input disturbance vector

The measurement process is described by the equation

$$\underline{z}(t) = H(t)\underline{x}(t) + \underline{v}(t) \tag{1.2}$$

where $\underline{z}(t)$ = measurement vector

$\underline{x}(t)$ = state variable vector

$\underline{v}(t)$ = measurement noise vector

The foundation of the maximum likelihood method is maximizing the conditional probability of the vector for unknowns given the measurements of the states.

$$\text{maximize } p(\underline{\theta}|z_n)$$

where p denotes the probability density

θ = vector of unknowns to be identified

z_n = sequence of measurements

Maximizing $p(\theta|z_n)$ is equivalent to maximizing the log likelihood function.

$$L = \ln[p(\underline{\theta}|z_n)] \tag{1.3}$$

where $L \triangleq$ log likelihood function

Sequential application of Baye's Rule to the conditional probability of θ given z_n yields:

$$\begin{aligned} p(\underline{\theta}|z_n) &= p(z_n|\underline{\theta})\ p(\underline{\theta}|z_{n-1}) && (1.4)\\ &= p(z_n|\underline{\theta})\ p(z_{n-1}|\underline{\theta})\ p(\underline{\theta}|z_{n-2})\\ &= p(z_n|\underline{\theta})\ p(z_{n-1}|\underline{\theta})\ p(z_{n-2}|\underline{\theta})\ p(\underline{\theta}|z_{n-3})\\ &= \prod_{i=1}^{N} p(z_i|\underline{\theta})\ p(\underline{\theta}|0) && (1.5)\end{aligned}$$

Because the state vector is dependent upon the vector of unknowns the equation

$$p(\underline{\theta}|z_n) = \prod_{i=1}^{N} p(z_i|\underline{\theta})\ p(\underline{\theta}|0)$$

can be re-written as

$$p(\underline{\theta}|z_n) = \prod_{i=1}^{N} p(z_i|\underline{x}_i(\underline{\theta}))\ p(\underline{\theta}|0) \tag{1.6}$$

Assuming the input disturbance ($\underline{w}(t)$) and the measurement noise ($\underline{v}(t)$) are gaussian, then the formula for multivariate gaussian conditional distribution can be applied because $p(\underline{z}_i|\underline{x}_i(\underline{\theta}))$ is gaussian.

$$p(\underline{z}_i|\underline{x}_i(\underline{\theta})) = \frac{1}{(2\pi)^{N/2}\ |B_i|^{1/2}} \cdot e^{-\frac{1}{2}(\underline{z}_i - E[\underline{z}_i])^T B_i^{-1}(\underline{z}_i - E[\underline{z}_i])} \tag{1.7}$$

In equation 1.7 $E[\underline{z}_i] = H_i\underline{\hat{x}}_i$ where $\underline{\hat{x}}$ is the output of a Kalman Filter. The matrix B_i is defined as follows:

$$B_i = E[(\underline{z}_i - E[\underline{z}_i])(\underline{z}_i - E[\underline{z}_i])^T]$$

$$B_i = E[(\underline{z}_i - H_i\underline{\hat{x}}_i)(\underline{z}_i - H_i\underline{\hat{x}}_i)^T]$$

$$B_i = H_i \; P_i \; H_i^T + R_i \tag{1.8}$$

Combining equations 1.3, 1.6 and 1.7 yields:

$$L = \ln\{[\prod_{i=1}^{N} \frac{1}{2\pi^{(N/2)} \; |B_i|^{(1/2)}} \cdot e^{-\frac{1}{2}(\underline{z}_i - H_i\underline{\hat{x}}_i)^T \; B_i^T(\underline{z}_i - H_i\underline{\hat{x}}_i)}] \cdot p(\underline{\theta}|0)\} \tag{1.9}$$

$$L = -\frac{1}{2} \sum_{i=1}^{N} \{(\underline{z}_i - H_i\underline{\hat{x}}_i)^T \; B_i^{-1}(\underline{z}_i - H_i\underline{\hat{x}}_i) + \ln|B_i|\}$$

$$- (\frac{N}{2})\ln(2\pi) + \ln[p(\underline{\theta}|0)] \tag{1.10}$$

$$L = -\frac{1}{2} \sum_{i=1}^{N} \{(\underline{z}_i - H_i\underline{\hat{x}}_i)^T \; B_i^{-1}(\underline{z}_i - H_i\underline{\hat{x}}_i) + \ln|B_i|\} + \text{constant} \tag{1.11}$$

The maximum likelihood problem involves maximizing equation 1.11 with respect to theta, subject to the constraints of the Kalman Filter equations. The Kalman Filter equations must be satisfied because the state estimate $\underline{\hat{x}}_i$ and the state covariance (P_i) are outputs of the Kalman Filter. A summary of the continuous time, time varying Kalman Filter equations are given below (5).

System Model:

$$\dot{\underline{x}}(t) = F(t)\underline{x}(t) + L(t)\underline{u}(t) + G(t)\underline{w}(t) \tag{1.12}$$

Measurement Model:

$$\underline{z}(t) = H(t)\underline{x}(t) + \underline{v}(t) \tag{1.13}$$

State Estimates:

$$\dot{\underline{\hat{x}}}(t) = F(t)\underline{\hat{x}}(t) + L(t)\underline{u}(t) + K(t) \; \underline{z}(t) - H(t)\underline{\hat{x}}(t) \tag{1.14}$$

Error Covariance Propagation:

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) - P(t)H^T(t)R^{-1}(t)H(t)P(t) \qquad (1.15)$$

Kalman Gain Matrix:

$$K(t) = P(t)H^T(t)R^{-1}(t) \qquad (1.16)$$

where $\underline{v}(t) \sim N(0,R(t))$ (1.17)

and $\underline{w}(t) \sim N(0,Q(t))$ (1.18)

The notation used in equation 1.17 denotes that $\underline{v}(t)$ is a random sequence with a gaussian probability density with a mean value and variance defined below.

$$E[\underline{v}(t)] = 0 \qquad (1.19)$$

$$E[(\underline{v}(t) - E[\underline{v}(t)])(\underline{v}(t) - E[\underline{v}(t)])^T] = R(t) \qquad (1.20)$$

Equation 1.14 is solved on the digital computer for discrete values of $\underline{\hat{x}}_i$ using equations 1.21 through 1.24.

$$\underline{\hat{x}}(t_o+\Delta t) = \Phi(t_o,\Delta t)\underline{\hat{x}}(t_o) + \Theta(t_o)\underline{z}(t_o) + \Gamma(t_o)\underline{u}(t_o) \qquad (1.21)$$

where $$\Phi(t,\Delta t) = e^{[F(t_o) - K(t_o)H(t_o)]\Delta t} \qquad (1.22)$$

$$\Theta(t_o) = \Phi(t_o,\Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1} \cdot [I - \Phi^{-1}(t_o,\Delta t)]K(t_o) \qquad (1.23)$$

$$\Gamma(t_o) = \Phi(t_o,\Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1} \cdot [I - \Phi^{-1}(t_o,\Delta t)]L(t_o) \qquad (1.24)$$

The error covariance propagation equation (1.15) is solved using the Kalman-Engler solution.

$$P(t_o+\Delta t) = [\phi_{\lambda y}(t_o,\Delta t) + \phi_{\lambda\lambda}(t_o,\Delta t)P(t_o)] \cdot [\phi_{yy}(t_o,\Delta t) + \phi_{y\lambda}(t_o,\Delta t)P(t_o)]^{-1} \qquad (1.25)$$

where

$$\begin{bmatrix} \underline{y}(t_o+\Delta t) \\ \underline{\lambda}(t_o+ t) \end{bmatrix} = \begin{bmatrix} \phi_{yy}(t_o,\Delta t) & \phi_{y\lambda}(t_o,\Delta t) \\ \phi_{\lambda y}(t_o,\Delta t) & \phi_{\lambda\lambda}(t_o,\Delta t) \end{bmatrix} \cdot \begin{bmatrix} \underline{y}(t_o) \\ \underline{\lambda}(t_o) \end{bmatrix} \qquad (1.26)$$

and

$$\Phi(t_o,\Delta t) = \begin{bmatrix} \phi_{yy}(t_o,\Delta t) & \phi_{y\lambda}(t_o,\Delta t) \\ \phi_{\lambda y}(t_o,\Delta t) & \phi_{\lambda\lambda}(t_o,\Delta t) \end{bmatrix} \tag{1.27}$$

The equations for $\dot{\underline{y}}(t)$ and $\dot{\underline{\lambda}}(t)$ are given in equation 1.28.

$$\begin{bmatrix} \dot{\underline{y}}(t) \\ \dot{\underline{\lambda}}(t) \end{bmatrix} = \begin{bmatrix} -F^T(t) & H^T(t)R^{-1}(t)H(t) \\ G(t)Q(t)G^T(t) & F(t) \end{bmatrix} \begin{bmatrix} \underline{y}(t) \\ \underline{\lambda}(t) \end{bmatrix} \tag{1.28}$$

Therefore

$$\Phi(t_o,\Delta t) = \exp \begin{bmatrix} -F^T(t_o) & H^T(t_o)R^{-1}(t_o)H(t_o) \\ G(t_o)Q(t_o)G^T(t_o) & F(t_o) \end{bmatrix} \Delta t \tag{1.29}$$

The linear system of equations in 1.29 is derived in Reference (3).

The likelihood function can be expanded into a Taylor series with only three terms because the likelihood function for Gaussian conditional probability distribution is quadratic.

$$L(\theta) = L(\theta_o) + \left.\frac{\partial L}{\partial \underline{\theta}}\right|_{\underline{\theta}_o} (\theta-\theta_o) + (\underline{\theta}-\theta_o)^T \left[\frac{1}{2}\left.\frac{\partial^2 L}{\partial \theta^2}\right|_{\underline{\theta}_o}\right] (\underline{\theta}-\underline{\theta}_o) \tag{1.30}$$

Taking the partial derivative of $L(\underline{\theta})$ with respect to $\underline{\theta}$ and setting it equal to zero to find the maximum:

$$\left.\frac{\partial L}{\partial \underline{\theta}}\right|_{\underline{\theta}^*} = \left.\frac{\partial L}{\partial \underline{\theta}}\right|_{\underline{\theta}_o} + \left.\frac{\partial^2 L}{\partial \underline{\theta}^2}\right|_{\underline{\theta}_o} (\underline{\theta}^*-\underline{\theta}_o) = 0 \tag{1.31}$$

where $\underline{\theta}^*$ is the value of $\underline{\theta}$ that results in the maximum of the likelihood function. Solving for $\underline{\theta}^*$ yields:

$$\underline{\theta}^* = \underline{\theta}_o - \left[\left.\frac{\partial^2 L}{\partial \underline{\theta}^2}\right|_{\underline{\theta}_o}\right]^{-1} \left[\left.\frac{\partial L}{\partial \underline{\theta}}\right|_{\underline{\theta}_o}\right]^T \tag{1.32}$$

In equation 1.32 $\Delta\theta$, or the step in theta is equal to:

$$\Delta\underline{\theta} = - \left[\frac{\partial^2 L}{\partial \underline{\theta}^2} \bigg|_{\underline{\theta}_o} \right]^{-1} \left[\frac{\partial L}{\partial \underline{\theta}} \bigg|_{\underline{\theta}_o} \right]^T \tag{1.33}$$

$\frac{\partial L}{\partial \underline{\theta}}$ is defined as the gradient and is equal to:

$$\frac{\partial L}{\partial \underline{\theta}} = \sum_{i=1}^{N} \nu^T(t_i) B^{-1}(t_i) \frac{\partial \nu(t_i)}{\partial \underline{\theta}}$$

$$- \frac{1}{2} \nu^T(t_i) B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \underline{\theta}} B^{-1}(t_i) \nu(t_i)$$

$$+ \frac{1}{2} \operatorname{tr} \left[B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \underline{\theta}} \right] \tag{1.34}$$

where

$$\nu(t_i) = \underline{z}(t_i) - H(t_i) \hat{\underline{x}}(t_i) \tag{1.35}$$

$$\frac{\partial \nu(t_i)}{\partial \underline{\theta}} = -H(t_i) \frac{\partial \hat{x}(t_i)}{\partial \underline{\theta}} - \frac{\partial H(t_i)}{\partial \underline{\theta}} \tag{1.36}$$

and

$$\frac{\partial B(t_i)}{\partial \underline{\theta}} = H(t_i) \left[P(t_i) \frac{\partial (H^T(t_i))}{\partial \underline{\theta}} + \frac{\partial P(t_i)}{\partial \underline{\theta}} H^T(t_i) \right]$$

$$+ \frac{\partial H(t_i)}{\partial \underline{\theta}} P(t_i) H^T(t_i) + \frac{\partial R(t_i)}{\partial \underline{\theta}} \tag{1.37}$$

The term $\frac{\partial^2 L}{\partial \underline{\theta}^2}$ is defined as the Fisher information matrix.

$$\frac{\partial^2 L}{\partial \underline{\theta}^2} \approx \sum_{i=1}^{N} \frac{\partial \nu^T(t_i)}{\partial \underline{\theta}} B^{-1}(t_i) \frac{\partial \nu(t_i)}{\partial \underline{\theta}}$$

$$- (2) \nu^T(t_i) B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \underline{\theta}} B^{-1}(t_i) \frac{\partial \nu(t_i)}{\partial \underline{\theta}}$$

$$- \frac{1}{2} \operatorname{tr} \left[B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \underline{\theta}} B^{-1}(t_i) \frac{\partial B(t_i)}{\partial \underline{\theta}} \right] \tag{1.38}$$

The term $\frac{\partial \hat{x}(t_i)}{\partial \underline{\theta}}$ in equation 1.36 is referred to as the sensitivity term and is derived from the Kalman Filter state estimate equation (1.14).

$$\frac{\partial\underline{\hat{x}}(t)}{\partial\underline{\theta}} = F(t)\,\frac{\partial\underline{\hat{x}}(t)}{\partial\underline{\theta}} + \frac{\partial F(t)}{\partial\theta}\,\underline{\hat{x}}(t) + \frac{\partial L(t)}{\partial\underline{\theta}}\,\underline{u}(t) - K(t)H(t)\,\frac{\partial\underline{\hat{x}}(t)}{\partial\underline{\theta}}$$

$$- K(t)\,\frac{\partial H(t)}{\partial\underline{\theta}}\,\underline{\hat{x}}(t) + \frac{\partial K(t)}{\partial\underline{\theta}}\,\underline{z}(t) - \frac{\partial K(t)}{\partial\underline{\theta}}\,H(t)\underline{\hat{x}}(t) \qquad (1.39)$$

See Appendix A for the derivation of equation 1.39.

Equation 1.39 can be solved for discrete values of $\frac{\partial\underline{x}(t)}{\partial\theta}$ on the digital computer using the following equations.

$$\frac{\partial\hat{x}(t_i)}{\partial\underline{\theta}}\ \underline{\Delta}\ \frac{\partial\underline{\hat{x}}(t_o+\Delta t)}{\partial\underline{\theta}} = \Phi(t_o,\ t)\,\frac{\partial\underline{\hat{x}}(t_o)}{\partial\underline{\theta}} + \Lambda(t_o)\underline{\hat{x}}(t_o)$$

$$+ \Psi(t_o)u(t_o) + T(t_o)\underline{z}(t_o) \qquad (1.40)$$

where

$$\Phi(t_o,\Delta t) = e^{[F(t_o)-K(t_o)H(t_o)]\Delta t} \qquad (1.41)$$

$$\Lambda(t_o) = \Phi(t_o,\Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1}$$

$$[I - \Phi^{-1}(t_o,\Delta t)]\left[\frac{\partial F(t_o)}{\partial\underline{\theta}} - K(t_o)\,\frac{\partial H(t_o)}{\partial\underline{\theta}} - \frac{\partial K(t_o)}{\partial\underline{\theta}}\,H(t_o)\right] \qquad (1.42)$$

$$\Psi(t_o) = \Phi(t_o,\Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1}\cdot$$

$$[I - \Phi^{-1}(t_o,\Delta t)]\left[\frac{\partial L(t_o)}{\partial\underline{\theta}}\right] \qquad (1.43)$$

$$T(t_o) = \Phi(t_o,\Delta t)[F(t_o) - K(t_o)H(t_o)]^{-1}\cdot$$

$$[I - \Phi^{-1}(t_o,\ t)]\left[\frac{\partial K(t_o)}{\partial\underline{\theta}}\right] \qquad (1.44)$$

The term $\frac{\partial P(t_i)}{\partial\underline{\theta}}$ in equation 1.37 is solved using equation 1.45.

$$\frac{\partial P(t_o+\ t)}{\partial\underline{\theta}} = \left[\phi_{\gamma y}(t_o,\Delta t) + \phi_{\gamma\gamma}(t_o,\Delta t)\,\frac{\partial P(t_o)}{\partial\underline{\theta}}\right]\cdot$$

$$\left[\phi_{yy}(t_o,\Delta t) + \phi_{y\gamma}(t_o,\Delta t)\,\frac{\partial P(t_o)}{\partial\underline{\theta}}\right]^{-1} \qquad (1.45)$$

Equation 1.45 is an iterative solution to $\frac{\partial P(t_o + t)}{\partial \underline{\theta}}$ and is derived in Appendix B.

The final unknown term in the sensitivity equation is $\frac{\partial K(t)}{\partial \underline{\theta}}$. This term is derived by differentiating equation 1.16 with respect to theta

$$\frac{\partial K(t)}{\partial \underline{\theta}} = P(t) \left[\frac{\partial H^T(t)}{\partial \underline{\theta}} R^{-1}(t) + H^T(t) \frac{\partial R^{-1}(t)}{\partial \underline{\theta}} \right] + \frac{\partial P(t)}{\partial \underline{\theta}} H^T(t) R^{-1}(t) \qquad (1.46)$$

1.2 Application of the Maximum Likelihood Equations to the Wheelset

The equations developed in the last section apply to any linear dynamic system. In this section those equations will be tailored to the dynamically scaled wheelset problem.

The wheelset equations of motion are given in equations 1.47 through 1.50.

$$\dot{\underline{x}}(t) = F\underline{x}(t) + G\underline{w}(t) \qquad (1.47)$$

where

$$\underline{F} = \begin{bmatrix} \frac{-2f_{22} + cV}{MV} & \frac{2f_{22}}{M} & \frac{-K_y}{M} \\ 0 & -\frac{K_\psi V}{2f_{11}\ell^2} & -\frac{\alpha V}{\ell r_o} \\ 1 & 0 & 0 \end{bmatrix} \qquad \underline{x} = \begin{bmatrix} \dot{y} \\ \psi \\ (y-\bar{\delta}) \end{bmatrix} \qquad (1.48)$$

$$\underline{G} = \begin{bmatrix} \frac{c}{M} \\ 0 \\ -1 \end{bmatrix} \qquad \underline{w} = \dot{\bar{\delta}} \qquad (1.49)$$

and

$$\underline{w}(t) \sim N(0,Q) \qquad (1.50)$$

See Reference (11) for a detailed development of the wheelset equations of motion. Appendix C contains definitions for the variable of the F and G matrix.

The wheelset measurement equation is given in 1.51

$$\underline{z}(t) = H\,\underline{x}(t) + \underline{v}(t) \tag{1.51}$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.52}$$

and

$$\underline{v}(t) \sim N(0,R) \tag{1.53}$$

The system dynamics matrix (F), the system random input matrix (G), and the system measurement matrix (H), the covariance matrix for the random input disturbance (Q) and the covariance matrix for the measurement noise (R) are all time-invariant.

The creep coefficients f_{11} and f_{22} in the system dynamics matrix are the parameters to be identified by the maximum likelihood method.

For the purposes of simplifying the maximum likelihood equations the actual creep coefficients will be defined as a multiple of their nominal values.

$$f_{22} = \beta_{22}\, f_{22_o} \tag{1.54}$$

$$f_{11} = \beta_{11}\, f_{11_o} \tag{1.55}$$

where, for simplicity

$$\beta_{22} = \beta_{11} = \beta \tag{1.56}$$

The nominal values for the creep coefficients have been identified in other research on railroad vehicle stability (4).

Combining equations 1.48 and 1.54 through 1.56 the system dynamics matrix is:

$$F = \begin{bmatrix} \dfrac{-2\beta f_{22_o} + cV}{MV} & \dfrac{2\beta f_{22_o}}{M} & \dfrac{-K_y}{M} \\ 0 & \dfrac{-K_\psi V}{2\beta f_{11_o}\,\ell^2} & \dfrac{-\alpha V}{\ell r_o} \\ 1 & 0 & 0 \end{bmatrix} \tag{1.57}$$

Since the system dynamics matrix contains the only parameter to be identified the vector of unknowns $\underline{\theta}$ reduces to a scalar.

$$\underline{\theta} = \beta \tag{1.58}$$

The maximum likelihood equations of the last section can be greatly simplified for two reasons:

1) the wheelset is a time-invariant system.

2) the system dynamics matrix (F) contains the only parameter to be identified.

A summary of the time-invariant Kalman Filter Equations for the wheelset system is given below

State Estimates:

$$\dot{\hat{\underline{x}}}(t) = F\hat{\underline{x}}(t) + K\underline{z}(t) - KH\hat{\underline{x}}(t) \tag{1.59}$$

Error Covariance Propagation:

$$\dot{P}(t) = FP(t) + P(t)F^T + GQG^T - P(t)\, H^TR^{-1}HP(t) \tag{1.60}$$

Kalman Gain Matrix:

$$K = P_{ss}H^TR^{-1} \tag{1.61}$$

$$\underline{V}(t) \sim N(0,R) \tag{1.62}$$

$$\underline{w}(t) \sim N(0,Q) \tag{1.63}$$

Equation 1.21, the solution to the Kalman filter estimates equation (1.14), reduces to

$$\hat{\underline{x}}(t_o+\Delta t) = \Phi(\Delta t)\ \hat{\underline{x}}(t_o) + \Theta\ \underline{z}(t_o) \tag{1.64}$$

where

$$\Phi(\Delta t) = e^{[F-KH]\Delta t} \tag{1.65}$$

$$\Theta = \Phi(\Delta t)[F-KH]^{-1} \cdot [I - \Phi^{-1}(\Delta t)]K \tag{1.66}$$

P_{ss} in equation 1.61 is the steady state solution to equation 1.67

$$P(t_o+\Delta t) = [\phi_{\lambda y}(\Delta t) + \phi_{\lambda\lambda}(\Delta t)\ P(t_o)] \cdot [\phi_{yy}(\Delta t) + \phi_{y\lambda}(\Delta t)\ P(t_o)]^{-1} \tag{1.67}$$

This equation is the same as equation 1.25 except that Φ is no longer dependent on t_o, but is a function only of Δt, as shown in equation 1.68.

$$\Phi(\Delta t) = \exp\begin{bmatrix} -F^T & H^T R^{-1} H \\ GQG^T & F \end{bmatrix} \Delta t \tag{1.68}$$

Because all of the elements of the matrix in equation 1.68 are time invariant, the iterative solution for P has to be solved only once. This value of P_{ss} is used to calculate the Kalman gain matrix according to equation 1.61. The Kalman gain matrix for a time invariant system such as the wheelset remains constant for the entire Kalman gain problem.

Taking into account the two simplifications listed above, the maximum likelihood equations reduce to:

$$L = -\frac{1}{2}\left[\sum_{i=1}^{N} \nu^T(t_i)B^{-1}\nu(t_i) + \ln|B|\right] \tag{1.69}$$

The constant term in equation 1.11 can be dropped because it shifts the likelihood function up or down but does not change the location of the maximum of the likelihood function.

$$\frac{\partial L}{\partial \underline{\theta}} = \sum_{i=1}^{N} \nu^T(t_i)B^{-1}\frac{\partial \nu(t_i)}{\partial \underline{\theta}} - \frac{1}{2}\nu^T(t_i)B^{-1}\frac{\partial B}{\partial \underline{\theta}}B^{-1}\nu(t_i) + \frac{1}{2}\operatorname{tr}\left[B^{-1}\frac{\partial B}{\partial \underline{\theta}}\right] \tag{1.70}$$

$$\frac{\partial \nu(t_i)}{\partial \underline{\theta}} = -H\frac{\partial \hat{x}(t_i)}{\partial \underline{\theta}} \tag{1.71}$$

$$\frac{\partial B}{\partial \underline{\theta}} = H\frac{\partial P}{\partial \underline{\theta}}H^T \tag{1.72}$$

$$\frac{\partial^2 L}{\partial \underline{\theta}^2} = \sum_{i=1}^{N} \frac{\partial \nu^T(t_i)B^{-1}}{\partial \underline{\theta}}\,\frac{\partial \nu(t_i)}{\partial \underline{\theta}} - (2)\nu^T(t_i)B^{-1}\frac{\partial B}{\partial \underline{\theta}}B^{-1}\frac{\partial \nu(t_i)}{\partial \underline{\theta}}$$

$$-\frac{1}{2}\operatorname{tr}\left[B^{-1}\frac{\partial B}{\partial \underline{\theta}}B^{-1}\frac{\partial B}{\partial \underline{\theta}}\right] \tag{1.73}$$

$$\frac{\partial \hat{\underline{x}}}{\partial \underline{\theta}} = F\frac{\partial \hat{\underline{x}}(t)}{\partial \underline{\theta}} + \frac{\partial F}{\partial \underline{\theta}}\hat{\underline{x}}(t) - KH\frac{\partial \hat{\underline{x}}(t)}{\partial \theta} - \frac{\partial K}{\partial \underline{\theta}}\underline{z}(t) - \frac{\partial K}{\partial \underline{\theta}}H\hat{\underline{x}}(t) \tag{1.74}$$

$$\frac{\partial \hat{\underline{x}}(t_i)}{\partial \underline{\theta}} = \frac{\partial \hat{\underline{x}}(t_o+\Delta t)}{\partial \underline{\theta}} = \Phi(\Delta t)\,\frac{\partial \hat{\underline{x}}(t_o)}{\partial \underline{\theta}} + \Lambda\,\hat{\underline{x}}(t_o) + T\,\underline{z}(t_o) \qquad (1.75)$$

where

$$\Phi(\Delta t) = e^{[F-KH]\Delta t} \qquad (1.76)$$

$$\Lambda = \Phi(\Delta t)[F-KH]^{-1} \cdot [I - \Phi^{-1}(\Delta t)][\frac{\partial F}{\partial \underline{\theta}} - \frac{\partial K}{\partial \underline{\theta}} H] \qquad (1.77)$$

$$T = \Phi(\Delta t)[F-KH]^{-1} \cdot [I - \Phi^{-1}(\Delta t)][\frac{\partial K}{\partial \underline{\theta}}] \qquad (1.73)$$

The term $\frac{\partial P}{\partial \underline{\theta}}$ is the steady state solution to equation 1.79.

$$\frac{\partial P(t_o+\Delta t)}{\partial \underline{\theta}} = [\phi_{\gamma y}(\Delta t) + \phi_{\gamma\gamma}(\Delta t)\,\frac{\partial P(t_o)}{\partial \underline{\theta}}] \cdot [\phi_{yy}(\Delta t) + \phi_{y\gamma}(\Delta t)\,\frac{\partial P(t_o)}{\partial \underline{\theta}}]^{-1} \qquad (1.79)$$

Equation 1.79 is a time invariant form of equation 1.45 and since $\frac{\partial P}{\partial \underline{\theta}}$ is time invariant for the wheelset problem, equation 1.79 is solved only once during each iteration of the maximum likelihood processor. Equation 1.79 is derived in Appendix D.

Equation 1.46 which defines $\frac{\partial K(t)}{\partial \underline{\theta}}$ reduces to:

$$\frac{\partial K}{\partial \underline{\theta}} = \left.\frac{\partial P}{\partial \underline{\theta}}\right|_{ss} H^T R^{-1} \qquad (1.80)$$

1.3 Maximum Likelihood Algorithm

The equations of the preceding section were implemented into a Fortran IV computer program. This section discusses the algorithm used to solve the wheelset maximum likelihood problem on the digital computer.

The maximum likelihood method is a batch processor in that the entire set of measurements is used for each iteration of the maximum likelihood equations (8). Each iteration of the maximum likelihood algorithm produces an estimate of the value of the parameter to be identified which will maximize the likelihood function given in equation 1.69. This estimate $\underline{\theta}^*$ then replaces $\underline{\theta}_o$ within the program and another iteration is made. When the following condition is met, iteration is stopped.

$$\underline{\theta}^* - \underline{\theta}_o = \Delta\underline{\theta} < \varepsilon \qquad (1.81)$$

During the development phase of a maximum likelihood processor it is helpful to calculate the value of the likelihood function (L), the gradient $(\frac{\partial L}{\partial \underline{\theta}})$, and the Fisher information matrix $(\frac{\partial^2 L}{\partial \theta^2})$ for a range of $\underline{\theta}$ where the maximum of the likelihood function is expected to lie. Changes in the flowchart of Figure 1.1 are shown in Figure 1.2. The flowchart at the bottom of the dotted box replaces the $\Delta\theta$ test at the end of the flowchart in Figure 1.1. This procedure is beneficial in that the maximum likelihood program can be checked out without the risk of an infinite loop being set up because of a convergence problem.

No mention has been made so far of the method used to calculate the state transition matrix (STM). On a digital computer, a very fast and accurate way to calculate the STM is through a power series expansion.

$$\Phi(\Delta t) = I + A\Delta t + \frac{1}{2!} A^2(\Delta t)^2 + \frac{1}{3!} A^3(\Delta t)^3 + \dots + \frac{A^i(\Delta t)^i}{i!} \qquad (1.82)$$

The series is truncated when the following condition is satisfied

$$\left(\sum_{i=1}^{N+1} A^i(\Delta t)^i \frac{1}{i!} + I \right) - \left(\sum_{i=1}^{N} A^i(\Delta t)^i \frac{1}{i!} + I \right) < \varepsilon \qquad (1.83)$$

where ε is some error criterion chosen by the user. A value of

$$\varepsilon = 1.0 \cdot 10^{-10} \qquad (1.84)$$

is usually considered to be sufficient. A listing of the maximum likelihood program is contained in Appendix E.

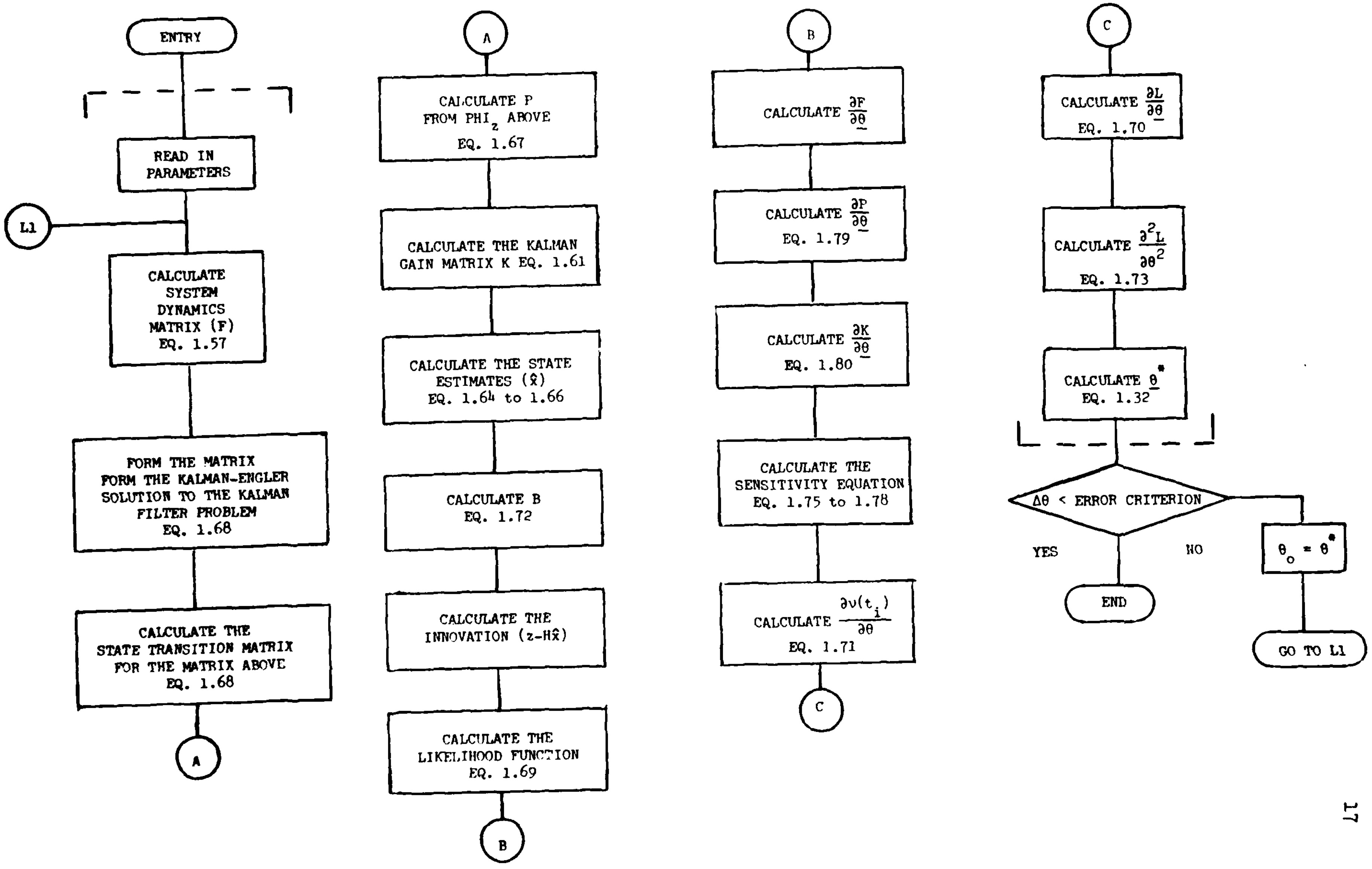

ENTRY
READ IN PARAMETERS
L1
CALCULATE SYSTEM DYNAMICS MATRIX (F) EQ. 1.57
FORM THE MATRIX FORM THE KALMAN-ENGLER SOLUTION TO THE KALMAN FILTER PROBLEM EQ. 1.68
CALCULATE THE STATE TRANSITION MATRIX FOR THE MATRIX ABOVE EQ. 1.68
A
A
CALCULATE P FROM PHI$_z$ ABOVE EQ. 1.67
CALCULATE THE KALMAN GAIN MATRIX K EQ. 1.61
CALCULATE THE STATE ESTIMATES ($\hat{x}$) EQ. 1.64 to 1.66
CALCULATE B EQ. 1.72
CALCULATE THE INNOVATION ($z-H\hat{x}$)
CALCULATE THE LIKELIHOOD FUNCTION EQ. 1.69
B
B
CALCULATE $\frac{\partial F}{\partial \underline{\theta}}$
CALCULATE $\frac{\partial P}{\partial \underline{\theta}}$ EQ. 1.79
CALCULATE $\frac{\partial K}{\partial \underline{\theta}}$ EQ. 1.80
CALCULATE THE SENSITIVITY EQUATION EQ. 1.75 to 1.78
CALCULATE $\frac{\partial \nu(t_i)}{\partial \theta}$ EQ. 1.71
C
C
CALCULATE $\frac{\partial L}{\partial \underline{\theta}}$ EQ. 1.70
CALCULATE $\frac{\partial^2 L}{\partial \theta^2}$ EQ. 1.73
CALCULATE $\underline{\theta}^*$ EQ. 1.32
$\Delta\theta$ < ERROR CRITERION
YES
NO
END
$\theta_o = \theta^*$
GO TO L1

Figure 1.1 Maximum Likelihood Algorithm (8).

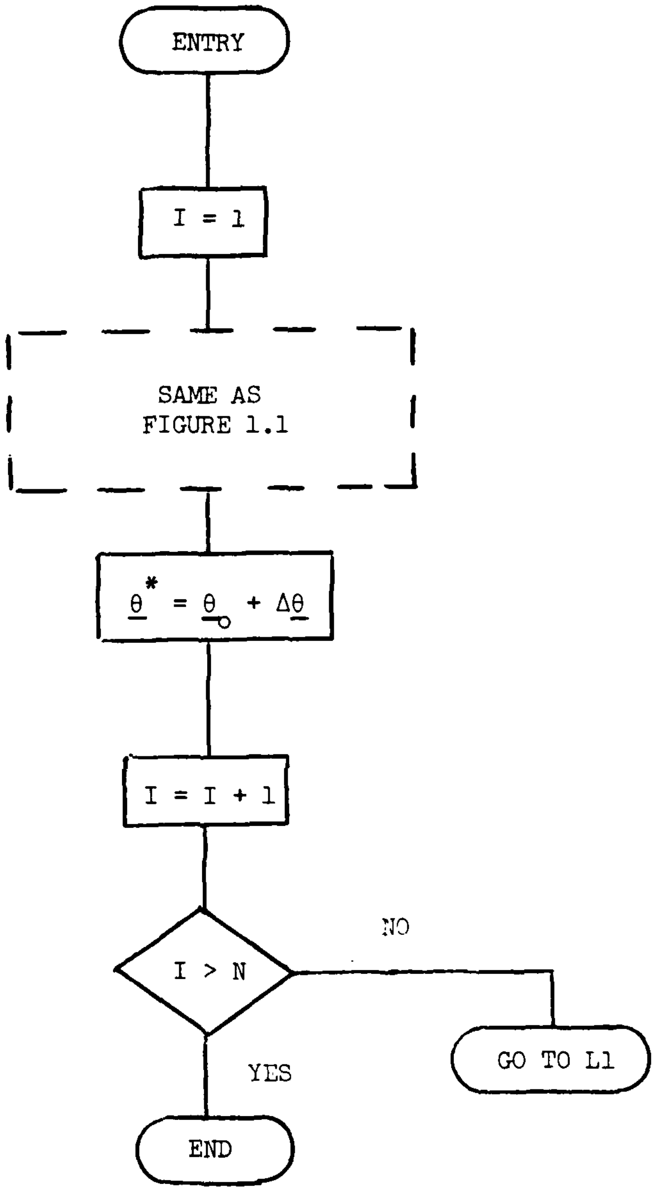

ENTRY
I = 1
SAME AS FIGURE 1.1
$\underline{\theta}^* = \underline{\theta}_o + \Delta\underline{\theta}$
I = I + 1
I > N
NO
GO TO L1
YES
END

Figure 1.2 Development Form of Maximum Likelihood Algorithm.

1.4 Test Data Generation

In order to ensure that the maximum likelihood computer program implement the maximum likelihood algorithm properly, it was tested using simulated wheelset data. The data was generated using the state transition matrix approach to solve the following equation.

$$\dot{\underline{x}} = F\underline{x} + G\underline{w} \qquad (1.85)$$

where F and G are specified by equations 1.57 and 1.49 respectively. Due to limitations in computer storage a total of 1000 data points for each state was all that could be generated.

The time step (Δt) was chosen by examining the eigenvalues of the F matrix. Typical values for the poles of the wheelset model are:

$$\sigma_1 = -210.7 \text{ sec}^{-1}$$

$$\sigma_{2,3} = -0.423 \pm j\ 5.982 \text{ sec}^{-1}$$

These poles indicate that transient responses are characterized by a lightly damped sinusoid ($\omega_n = 6.0 \text{ sec}^{-1}$; $\zeta = 0.07$) plus a rapidly converging exponential decay ($\tau = \frac{1}{\sigma_1} = 4.7$ msec). In the frequency domain a sharp resonant peak occurs at the natural frequency, with an additional breakpoint at $\omega = 1/\tau$.

As the estimates of the creep coefficients change, the damping ratio of the low frequency resonance changes, as does the location of the higher frequency real pole. In selecting the time interval ΔT and the record length $N\Delta T$ it is desirable for information on both high and low frequency modes to be present in the data. To obtain six data points per cycle at the higher frequency $f = \sigma_1/2\pi$ a ΔT is chosen to be 0.005 sec. Since computer storage constraints limited the record length to N = 1000 points, the record length is 5 seconds, which corresponds to 4.8 cycles of the low frequency mode.

Large values for Δt were tried but the state transition matrix approach used in the maximum likelihood program had convergence problems. A $\Delta t = .005$ was the largest time step that could be used without causing numerical problems.

In addition to the random track input $\underline{w}$ in equation 1.85, there was also measurement noise added to the state vectors.

$$\underline{z} = H\underline{x} + \underline{v} \tag{1.86}$$

$\underline{w}$ and $\underline{v}$ were generated by summing 10 random number vectors which were generated by an APL* operator. The random vectors generated by successive runs of this APL operator are not correlated. Ten of these random vectors were summed to force the distribution of $\underline{u}$ and $\underline{w}$ to be approximately Gaussian ($\underline{1}$). Once these gaussian random vectors were formed they could be manipulated to force their means and mean squared values to equal what the user desired.

Although maximum likelihood theory requires $\underline{w}$ and $\underline{v}$ to be white noise because these vectors were of finite length they were bandwidth limited. Also since only 10 random vectors were summed, the probability distributions of $\underline{w}$ and $\underline{v}$ were not exactly gaussian, only approximately gaussian. A listing of the APL functions used to generate the simulated wheelset data is contained in Appendix F.

* APL: A Programming Language. Implemented on the Princeton University IBM 370/158 time-sharing system.

Chapter 2

EXPERIMENTAL PROGRAM

2.1 Objective

The experimental program was designed to make measurements of the wheelset state variables. The third order wheelset model has the following state variables:

$\dot{y}$ = lateral velocity

ψ = yaw angle

$(y-\overline{\delta})$ = wheelset to rail centerline relative lateral displacement

See Reference (11) for a detailed development of the wheelset equations of motion.

2.2 Experimental Facility

The entire experimental program was carried out on the Princeton Dynamic Model Track. Previous research at the Dynamic Model Track involved the development and validation of the dynamically scaled wheelset model and the measurement, restraint and propulsion system for the wheelset (9).

The experimental apparatus consisted of the wheelset model and the idler carriage. Both of these components rode on the 400 foot LEXAN track. The idler carriage surrounded the wheelset but was dynamically isolated from it. The idler carriage provided restraints to prevent the wheelset from completely derailing. The idler carriage was also part of the linkage that connects the wheelset to the propulsion unit. Figures 2.1 and 2.2 diagram the wheelset-idler carriage relationship.

The propulsion system was a hydraulically operated drive unit that rode along a steel I-beam guideway above the LEXAN track. The hydraulic drive unit had a feedback control system for maintaining a constant, user-selected velocity.

Suspended from the propulsion unit on instrument racks were the on-board analog computer, the bridge amplifiers, and the portable four track FM analog cassette recorder. The bridge amplifiers were used to amplify the output of the lateral accelerometer.

2.3 Transducers

Figure 2.3 is a diagram of transducer placement on the wheelset and Figure 2.4 traces the output of the transducers through the signal conditioning equipment to the recorder.

Lateral velocity could not be measured directly, therefore an accelerometer was used to measure lateral acceleration. A bridge amplifier was used to excite the differential type accelerometer and to amplify the output of the accelerometer. The lateral acceleration signal will have to be integrated on a digital computer to obtain lateral velocity.

The yaw angle was measured directly using a potentiometer. The equation relating potentiometer output (ν) to the yaw angle (ψ) is:

$$\psi = m\nu + b \tag{2.1}$$

where m is the slope of the straight line described by this function and b is the y-intercept. The intercept b varied from dav to day due to amplifier drift. A pre-run and post-run procedure was developed so that the value of b could be determined at the start of every day.

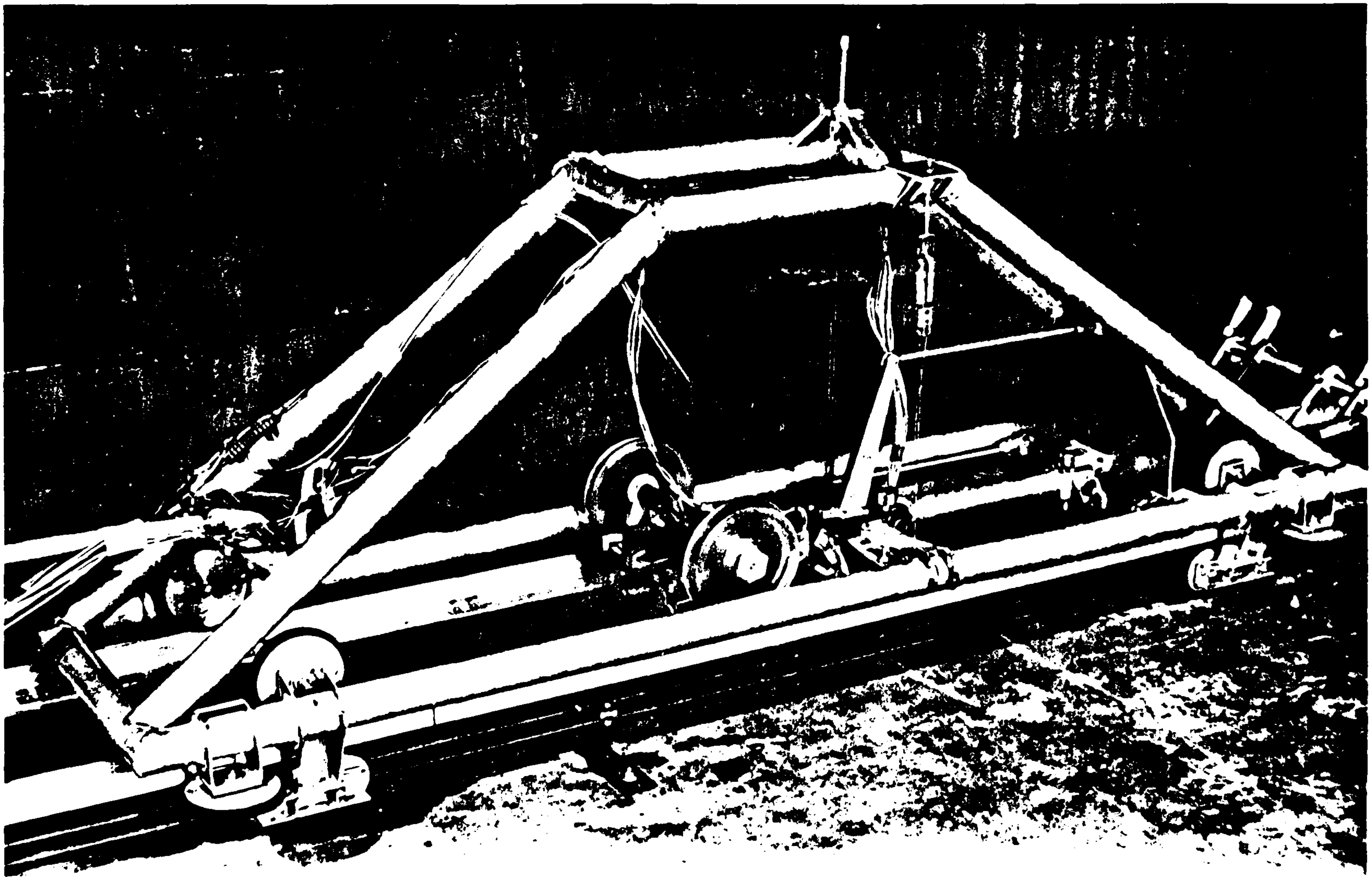


Figure 2.1a Side View of Wheelset Idler-Carriage Relationship

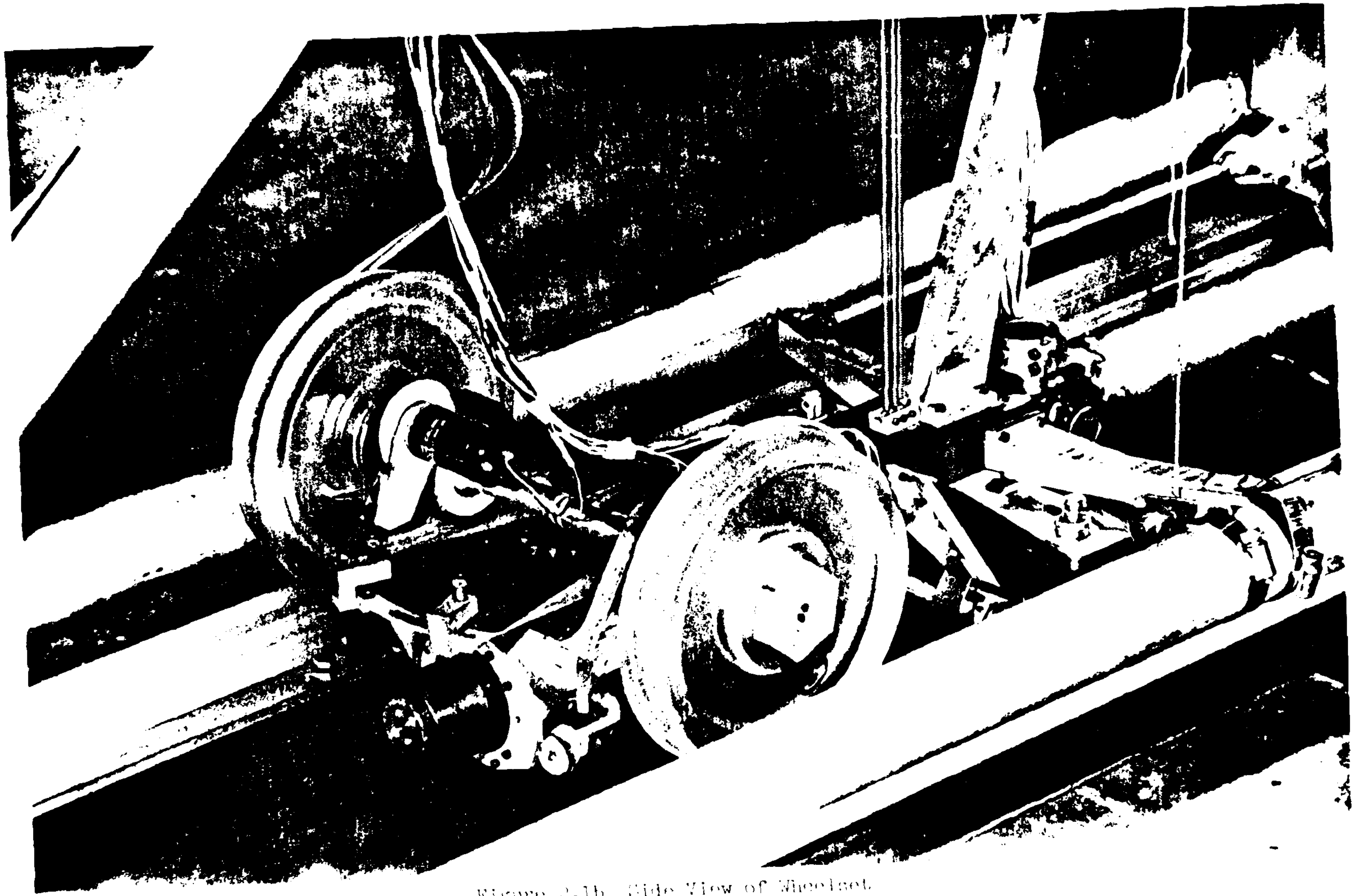


Figure 2.1b Side View of Wheelset
Idler-Carriage Relationship

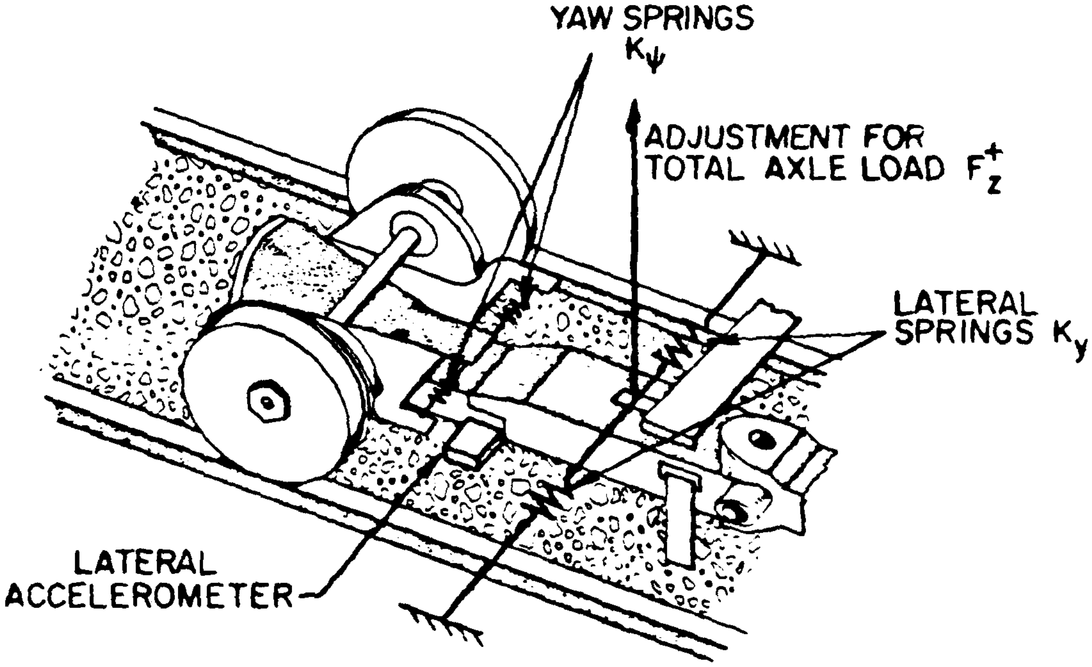


Figure 2.2 Top View of Wheelset Idler-Carriage Relationship.

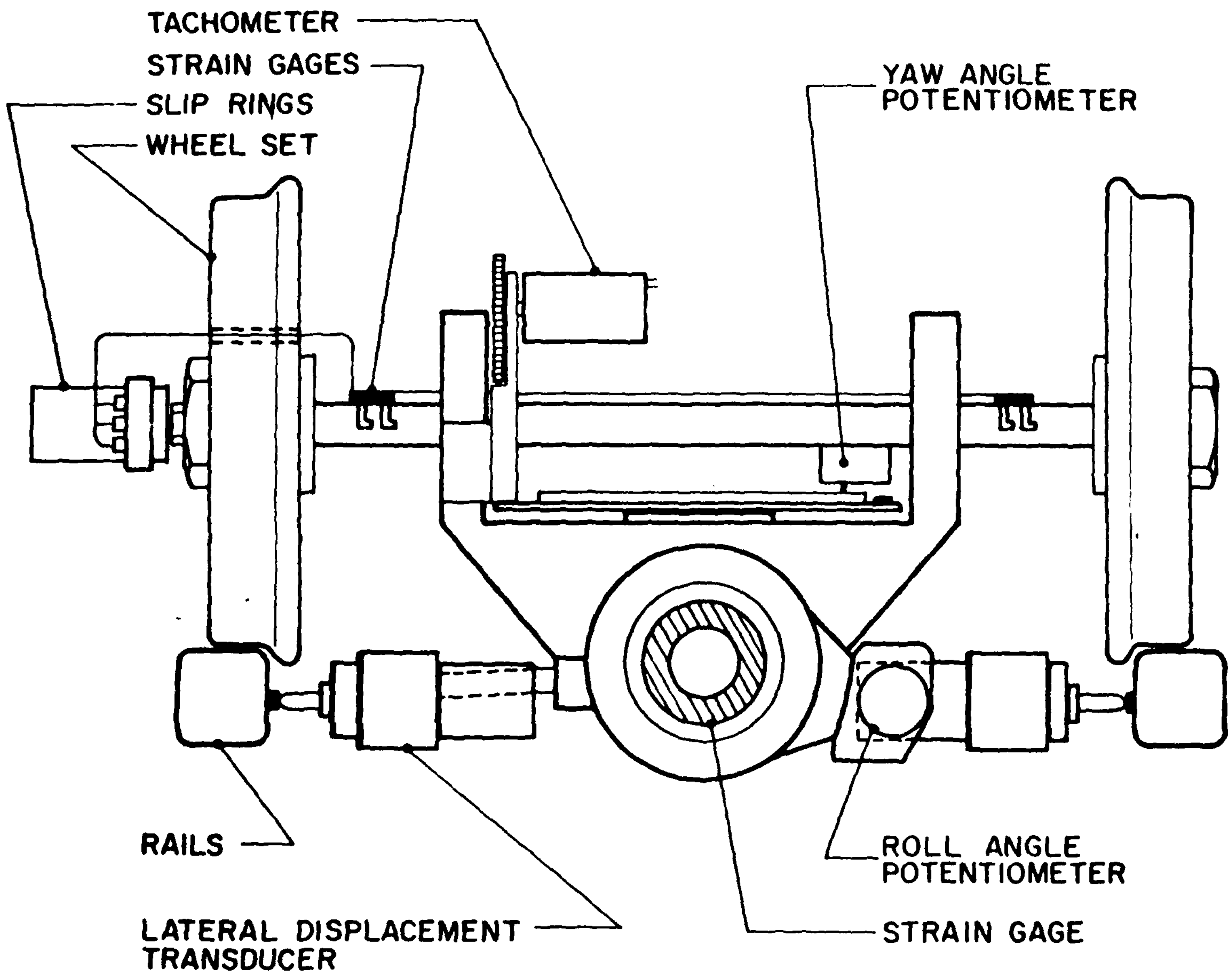


Figure 2.3 Wheelset Instrumentation.

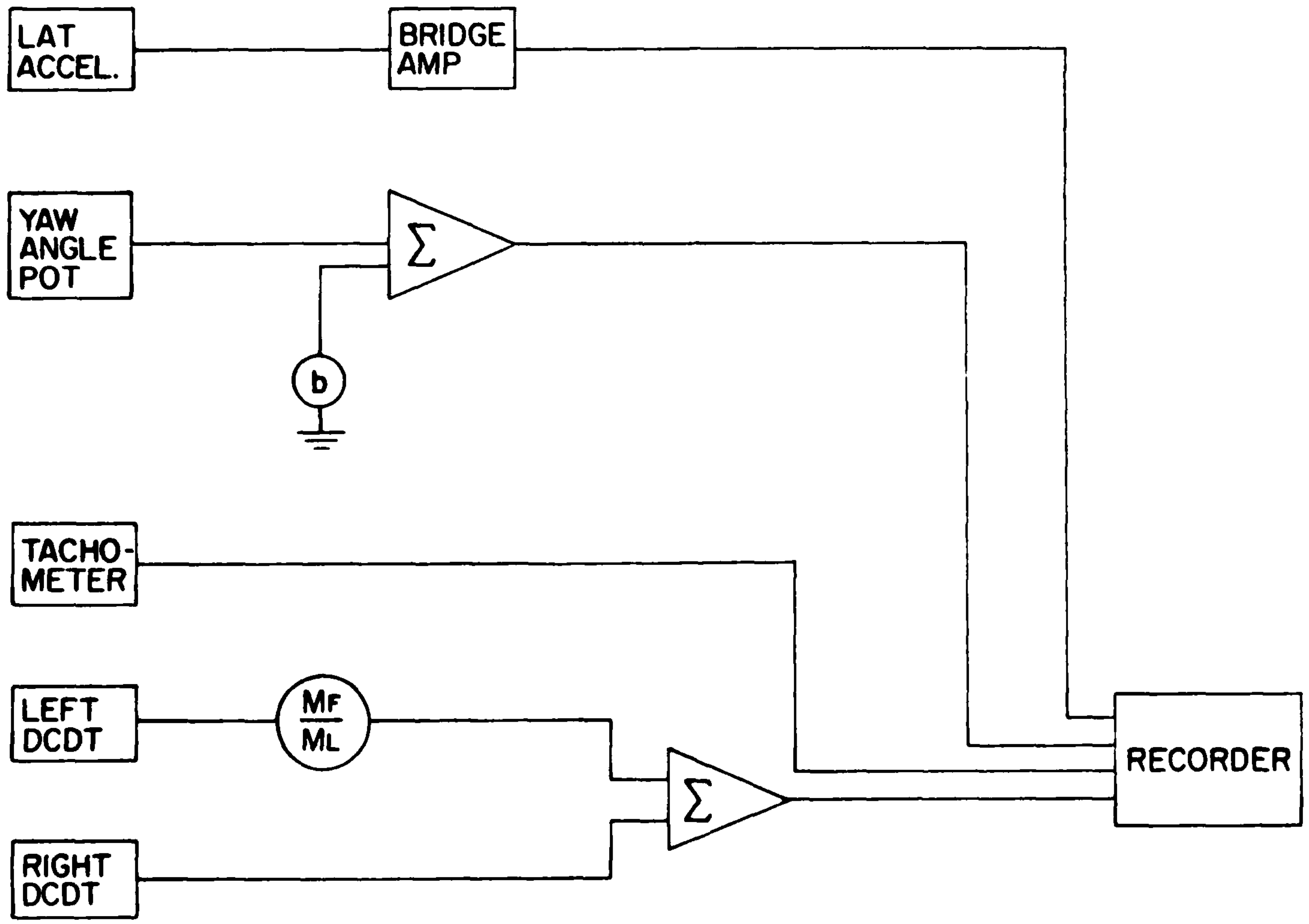


Figure 2.4 Transducer signal conditioning.

The variations in yaw angle during a run were smaller in magnitude than the value of the y-intercept. In order to obtain the best resolution on the cassette tape an electrical bias was summed with the yaw potentiometer output on the on-board analog computer. The value of the electrical bias was adjusted from day to day so that the output of the analog computer circuit equalled zero when the yaw angle was zero. The equation that describes the analog computer output is:

$$\eta = \frac{\psi - b}{m} + e \qquad (2.2)$$

where η = the output of the analog computer

e = the electrical bias

A displacement transducer (DCDT) was mounted on each side of the wheelset. Each displacement transducer measured the distance of the wheelset from the inside edge of the track. Figure 2.5 diagrams the relationship between the variable measured by the DCDT's and the state variable $(y-\overline{\delta})$. The equations for the DCDT's are:

$$V_r = m_r \, (y-\delta_2) \qquad (2.3)$$

$$V_L = m_L \, (y-\delta_1) \qquad (2.4)$$

where V_r = voltage output of the right DCDT

V_L = voltage output of the left DCDT

m_r = slope of the straight line function

m_L = slope of the straight line function

Multiplying V_L by $\frac{m_r}{m_L}$ and adding V_r to V_L:

$$V_r + \frac{m_r}{m_L} V_L = m_r(y-\delta_r) + m_r(y-\delta_L) \qquad (2.5)$$

This is the signal that was recorded during the experiment. The on-board

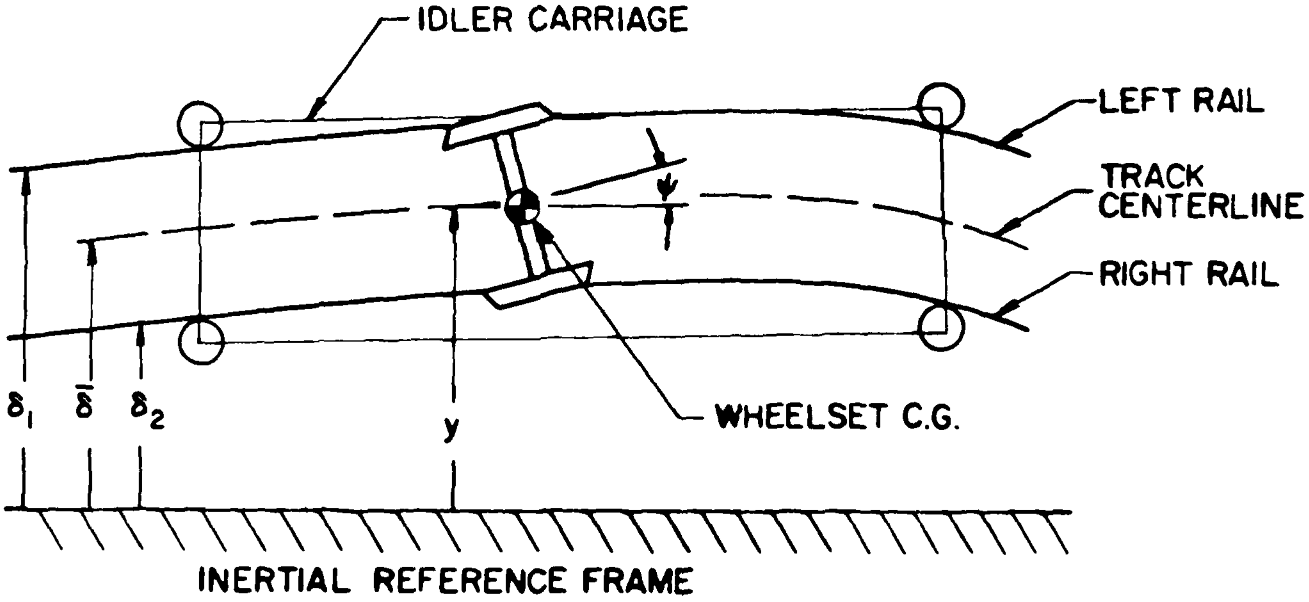


Figure 2.5 Definition of Lateral Displacement.

analog computer was used to multiply the left DCDT signal by $\frac{m_r}{m_L}$. The state variable $(y-\overline{\delta})$ can be derived from the recorded signal by dividing $V_r + \frac{m_r}{m_L} V_L$ by $2m_r$.

$$\frac{V_r + \frac{m_r}{m_L} V_L}{2\, m_r} = \frac{2\, m_r\, y - m_r\, (\delta_2 + \delta_1)}{2m_r} \tag{2.6}$$

$$(y-\overline{\delta}) = y - \frac{(\delta_1 + \delta_2)}{2} = y - \overline{\delta} \tag{2.7}$$

The DCDT's have a linear range of operation which is much smaller than their total range of operation. The linear range of operation is slightly different for each DCDT as shown in Figures 2.6 and 2.7. Each DCDT was positioned so that its output was the center of its linear range when the wheelset was centered on the track. Because the DCDT's were not mounted symmetrically about the wheelset's longitudinal centerline, a bias was introduced into the measuring process.

$$V_L = m_L\, (y-\delta_L) + b \tag{2.8}$$

$$V_r = m_r\, (y-\delta_r) \tag{2.9}$$

where b = position bias

The same algebraic operations as previously yield:

$$\frac{m_r}{m_L} V_L = \frac{m_r}{m_L} m_L\, (y-\delta_L) + \frac{m_r}{m_L} b \tag{2.10}$$

$$V_r = m_r\, (y-\delta_r) \tag{2.11}$$

$$V_r + \frac{m_r}{m_L} V_L = m_r\, (y-\delta_L) + \frac{m_r}{m_L} b + m_r\, (y-\delta_r) \tag{2.12}$$

$$= 2m_r y - m_r\, (\delta_L + \delta_r) + \frac{m_r}{m_L} b \tag{2.13}$$

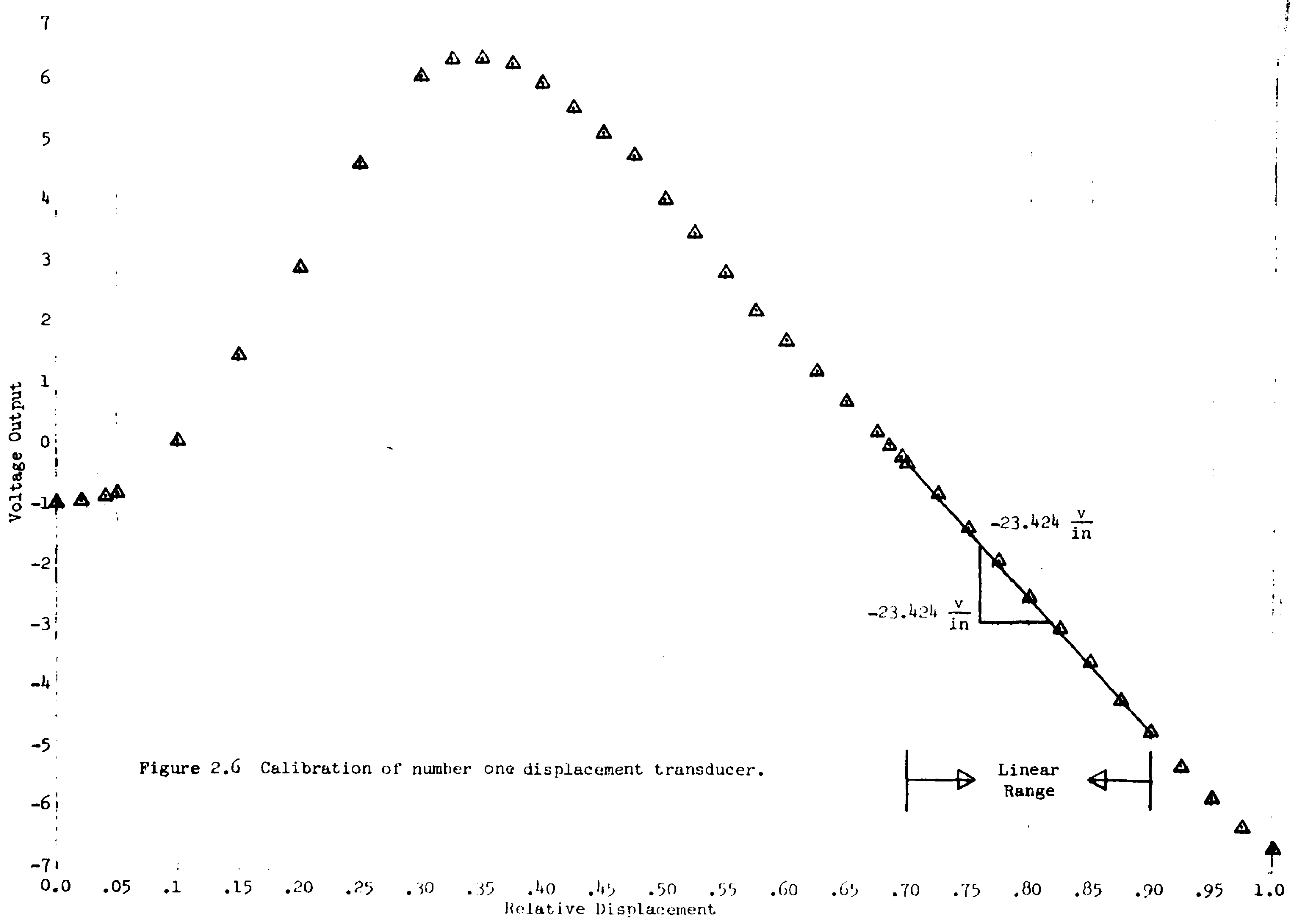


Figure 2.6 Calibration of number one displacement transducer.

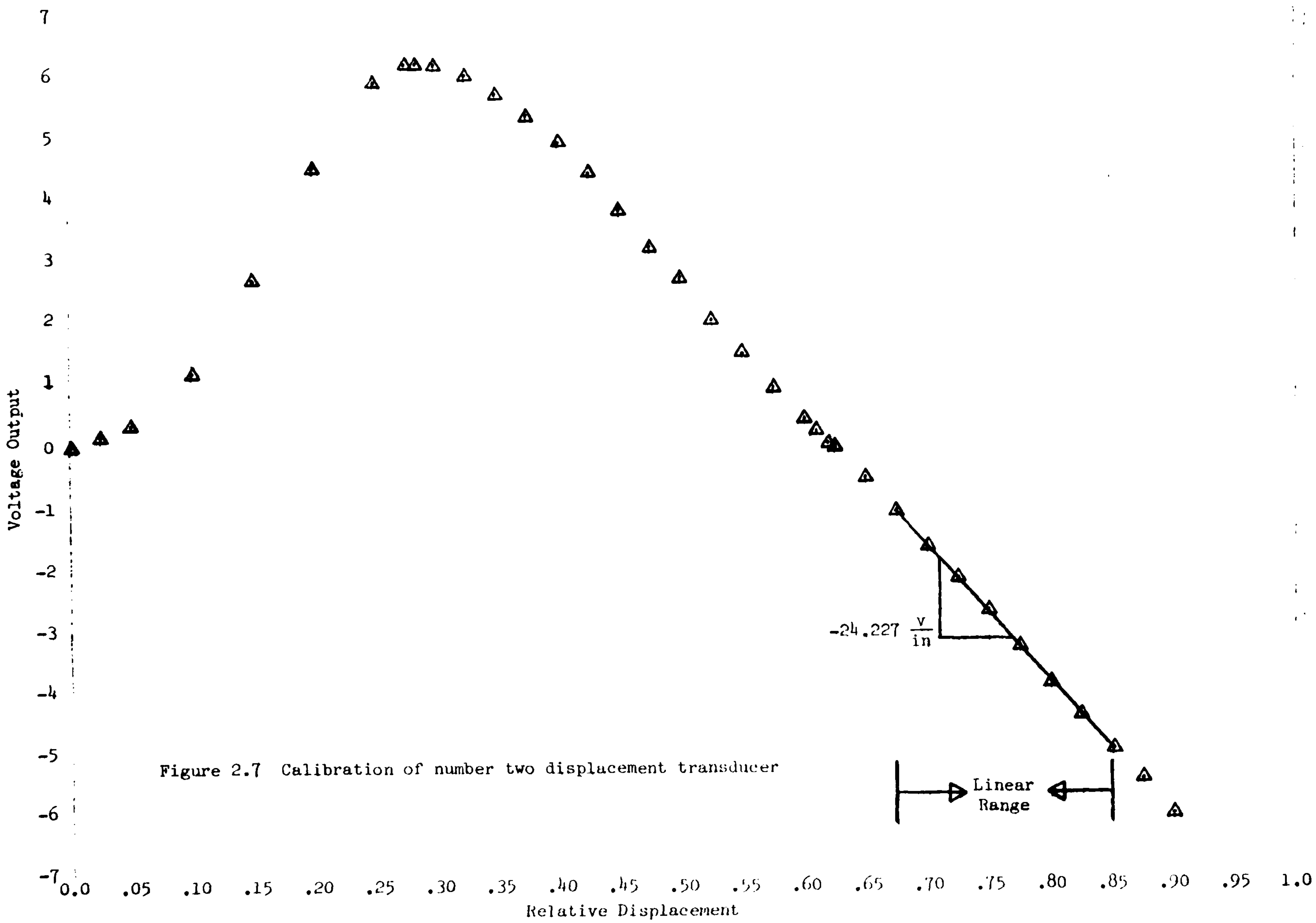


Figure 2.7 Calibration of number two displacement transducer

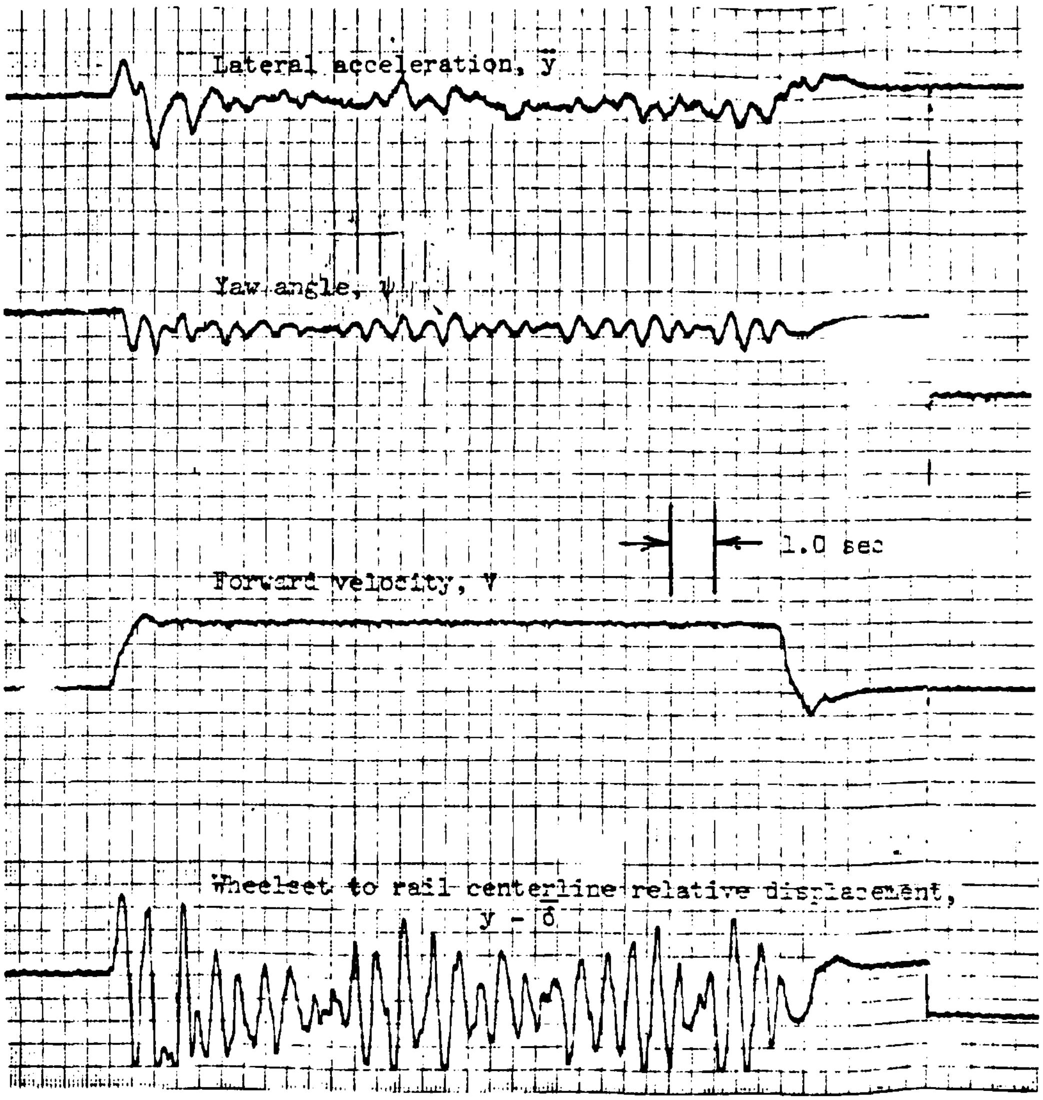


Figure 2.8 Typical transient response of wheelset to track inputs. Wheelset has lateral and yaw linear spring suspensions.

$$\frac{\dfrac{V_r + m_r V_L}{m_L}}{2m_r} = y - \frac{(\delta_L + \delta_r)}{2} + \frac{b}{2m_L} \qquad (2.14)$$

$$= (y - \bar{\delta}) + \frac{b}{2m_L} \qquad (2.15)$$

In addition to the three variables mentioned above, the wheelset axle angular velocity ($\ell = v/r_o$) in the rolling direction was also measured. Although the angular velocity is not a state variable, it is necessary to construct the velocity dependent state matrix F. The output of the tachometer was recorded directly with no processing on the on-board analog computer as with the lateral displacement and yaw angle signals. Sample responses from the above transducers are given in Figure 2.8.

2.4 Experimental Procedure

All of the above mentioned signals were recorded on a portable, four track FM analog cassette type recorder* which was carried on board the drive unit. The recorder was started and stopped manually before and after each run.

The bridge amplifier that powered the accelerometer was balanced at the beginning of each day to eliminate the drift in amplifier output that occurred from day to day.

The on-board analog computer had a built-in calibration circuit for the accelerometer bridge amplifier. This calibration circuit produced a known voltage that appeared to the accelerometer as an acceleration load. This acceleration load remained constant from day to day. In order to determine the input-output relationship of the accelerometer, the accelerometer output

* Philips Mini Log 4 Portable Analog Cassette Recorder.

was recorded during the calibration. This calibration test was performed before every data run to account for amplifier drift between data runs.

2.5 Measurement Noise

There were two sources of uncertainty in the data. The first of these was electrical noise. The bridge amplifier, analog computer and hydraulic pump for the propulsion unit all operate on 400 cycle alternating current.

Vibration of the cassette recorder during data runs was another source of noisy data. The steel I-beam guideway that supported the hydraulic drive unit had gaps between the I-beams. The vibration that resulted when the drive unit wheels hit these gaps was transmitted to the recorder, and showed up as noise on the data tapes. As mentioned earlier, the recorder was suspended from the drive unit on an instrument rack which had shock absorbers to help isolate the recorder from vibration. Nonetheless, some vibrations still affected the recorder and added noise to the data.

In addition to the two definite sources of noise, there is a third possible source of noise. It is known that there are small gaps between lengths of LEXAN rail. These small gaps are the result of contraction and expansion of the LEXAN material due to temperature changes. The 800 foot building which houses the Princeton Dynamic Model Track does not have a completely controlled environment; therefore, temperature variations of as much as 50°F are possible. Although no quantitative analysis has been carried out to verify this hypotheses, it is possible for the wheelset structural modes to be excited when the wheels hit these gaps. The transducers mounted on the wheelset, especially the lateral accelerometer, could detect this vibration, thereby adding noise to the data.

Finally to insure the validity of the data, the LEXAN track and wheels were cleaned before every run. Dirt and dust on the track or wheels could alter rail/wheel adhesion, in which case the wheelset model would be invalid. Absolute methanol was used as the cleaning agent because it left no residue on the track.

2.6 Filtering

To remove the noise due to the electrical system and vibration of the wheelset structural modes all of the signals were filtered off-line. The acceleration signal was filtered with a second order low-pass filter. The yaw angle and lateral displacement signals were filtered using seventh or eighth order band-pass filters, and the rotational velocity signal was filtered using a first order low-pass filter. The cutoff frequencies for the filters are given in Table 2.1.

Table 2.1

Filter Cutoff Frequencies

Signal	Filter Type	High Pass Cutoff	Low Pass Cutoff
Lateral Acceleration	low-pass	N/A	49.9 Hz
Yaw Angle	band-pass	.1 Hz	60 Hz
Rolling Angular Velocity	low-pass	N/A	20 Hz
Wheelset to Rail Relative Lateral Displacement	band-pass	.1 Hz	60 Hz

2.7 Digitization

After filtering the data had to be digitized so it could be stored and analyzed on a digital computer. As mentioned in the previous section, the lowest permissible sampling rate was determined when the low-pass filter cutoff frequencies were chosen. It was decided to use a sampling rate of 120 samples/second per channel. The A/D converter used for digitization was a 12 bit sample and hold A/D converter.* This A/D converter was controlled by a minicomputer system.** The minicomputer had two memories for storing data (memory A and memory B) each of which had the capacity to store 512 data points. When the digitizing process started, the computer stored the digitized data in memory A. Once memory A was full the computer began filling memory B while at the same time writing the data in memory A to magnetic tape. Since the A/D converter was a sample and hold type, there was only a several nanosecond time delay between the sample for each channel. However, the switching process from memory A to memory B took 3 msec., therefore the sampling interval between the 512th data point and the 513th data point was 11.33 msec. The sampling interval between all other sets of four data points was 8.33 msec. Figure 2.8 depicts this shift in sampling interval. Appendix G is a listing of the program that controlled the A/D converter during the digitizing process.

* Preston GMAD-1 Analog to Digital Conversion System.

** Hewlett Packard HP 1000 System.

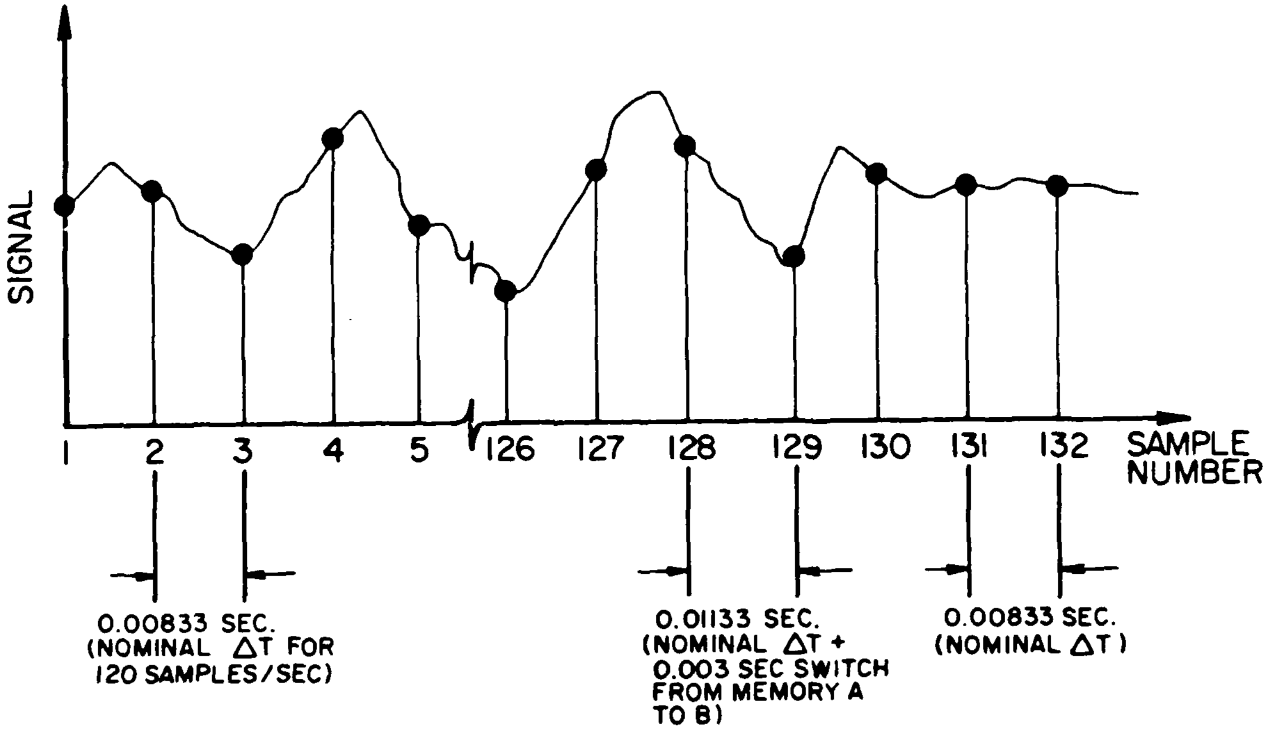

SIGNAL
1
2
3
4
5
126
127
128
129
130
131
132
SAMPLE NUMBER
0.00833 SEC. (NOMINAL ΔT FOR 120 SAMPLES/SEC)
0.01133 SEC. (NOMINAL ΔT + 0.003 SEC SWITCH FROM MEMORY A TO B)
0.00833 SEC. (NOMINAL ΔT)

Figure 2.9 Effect of memory switching on sampling rate.

Chapter 3

RESULTS

3.1 Outline of Test Cases

In this chapter the results obtained from the maximum likelihood computer program using the simulated data will be presented. As mentioned in Chapter 1, the APL program which generated the simulated data had provisions for any combination of deterministic input, random input, initial condition and measurement noise. Table 3.1 is a summary of the maximum likelihood program testing.

3.1.1 Explanation of Test Case Data

The LEXAN track lateral alignment is ideally a white noise velocity input, with a gaussian probability density. Because of these characteristics the track input does not have to be measured but can be described as a random input into the wheelset model. In this case, it is the Q matrix which gives the maximum likelihood processor all of its information about the track input. It is also possible to measure the track input, although there would be some uncertainty involved. If the track input were measured, then the measured values for the track input would become a deterministic input and the uncertainty in the track measurements would be described as a random track input.

For the first seven test cases the track input is treated as a random track input, with the only information describing it contained in the Q matrix.

In test cases 7 and 8 the track input is treated as a deterministic input. The random track input for these cases represents the uncertainty in the deterministic track input.

Table 3.1

Summary of Maximum Likelihood Program Testing

Test Case	Deterministic Track Input (u)	Initial Condition*	Random Track Input (w)		Measurement Noise (v)		
			Mean	Mean Squared Value	Mean	Mean Squared Value	
1	0	x(1)=x(2)=x(3)=10	0	0	0	0	Group 1
2	0	x(1)=x(2)=x(3)=0	0	1.0	0	0	
3	0	x(1)=x(2)=x(3)=10	0	0	0	.01	
4	0	x(1)=x(2)=x(3)=0	0	1.0	0	.01	
5	0	x(1)=x(2)=x(3)=10	0	1.0	0	.01	
6	0	x(1)=x(2)=x(3)=0	0	100.0	0	1.0	Group 2
7	0	x(1)=x(2)=x(3)=0	0	100.0	0	.01	
8	$E[u] = 0$ $E[u^2] = 100$	x(1)=x(2)=x(3)=0	0	.1	0	.1	Group 3
9	$E[u] = 0$ $E[u^2] = 100$	x(1)=x(2)=x(3)=0	0	10.0	0	.1	

$$* \quad \underline{x} = \begin{bmatrix} \dot{y} \\ \psi \\ (y-\bar{\delta}) \end{bmatrix}$$

In terms of generating the simulated wheelset data, the two representations of the track input are handled in the following way. For the case when the track input is considered to be a totally random input, a random vector is used as the input to the system model which generates the data. When this data is processed, the maximum likelihood program (specifically the Kalman Filter) is not told what the random vector was, the only information it is given is the covariance matrix for the vector Q.

When the track input is considered to be deterministic, the system model which generates the data has two random vectors as inputs. One of these random vectors represents the deterministic input;the other represents the uncertainty in the deterministic input. This time when the data generated by the system model are analyzed by the maximum likelihood processor, the Kalman Filter is given the vector that represents the deterministic input, but it is given only the covariance matrix for the vector that represents the uncertainty in the deterministic input. Figure 3.1 diagrams the generation of the simulated data for both representations of the track input. This figure is not designed to represent the entire process for generating simulated data (there can be measurement noise) nor is it designed to represent all of the inputs that go into the maximum likelihood processor. Its only purpose is to demonstrate the variations in representing the track input.

3.1.2 Purpose of Each Test Case

As stated previously, test cases 8 and 9 are different from the other seven test cases in how the track input is represented. Test case 9 is different from test case 8 in that the level of uncertainty in the deterministic track input is much higher. These two cases give some indication of how well the real track input would have to be measured to significantly increase the

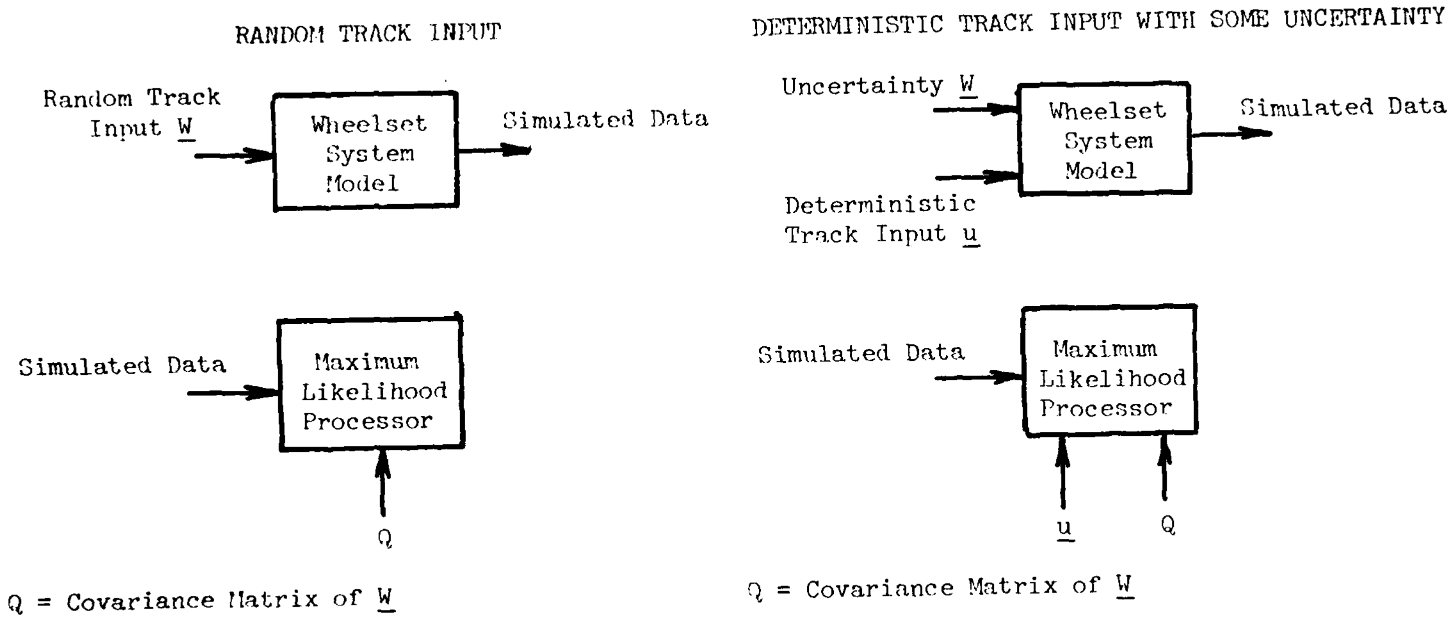

RANDOM TRACK INPUT
Random Track Input W
Wheelset System Model
Simulated Data
Simulated Data
Maximum Likelihood Processor
Q
Q = Covariance Matrix of W
DETERMINISTIC TRACK INPUT WITH SOME UNCERTAINTY
Uncertainty W
Deterministic Track Input u
Wheelset System Model
Simulated Data
Simulated Data
Maximum Likelihood Processor
u
Q
Q = Covariance Matrix of W

Figure 3.1 Representations of Track Input.

performance of the maximum likelihood processor over the case when the track is considered to be a random track input.

Test cases 1 through 5 were run to find out how various combinations of initial condition, random track input and measurement noise affect the maximum likelihood processor. Test case 1 through 3 were also run to find out how well that maximum likelihood processor performs when it is given incorrect information about the random track input and the measurement noise. In other words the simulated data for test cases 1 through 3 was generated with a random track input and measurement noise that had statistics as given in Table 3.1. However, when this data was analyzed using the maximum likelihood processor, the Kalman Filter was given incorrect information about the statistics of the $\underline{w}$ and $\underline{v}$ used to generate the data. This incorrect information took the form of an incorrect Q and R matrix. Table 3.2 shows the true Q and R matrices for $\underline{w}$ and $\underline{v}$ versus the Q and R matrices actually used by the Kalman Filter.

3.2 Method of Testing the Maximum Likelihood Parameter

Essentially the maximum likelihood program has 3 main parts. The first of these calculates the likelihood function based on the equation

$$L = -\frac{1}{2} \sum_{i=1}^{N} (\nu^T(t_i)B^{-1}\nu(t_i) + \ln |B|) \qquad (3.1)$$

The likelihood function has two main components as can be seen in equation 3.1.

$$\xi_{\text{observation}} = -\frac{1}{2} \sum_{i=1}^{N} \nu^T(t_i)B^{-1}\nu(t_i) \qquad (3.2)$$

and

$$\xi_{\text{bias}} = -\frac{1}{2} \sum_{i=1}^{N} \ln |B| \qquad (3.3)$$

Table 3.2

Q and R Matrices for Test Cases 1 Through 5

Test	Random Track Input $\underline{w}$				Measurement Noise $\underline{v}$			
Case	Mean	Mean Squared Value	Correct Value of Q for $\underline{w}$	Value of Q Actually Given to Maximum Likelihood Program	Mean	Mean Squared Value	Correct R Matrix for $\underline{v}$	R Matrix Actually Given to Maximum Likelihood Processor
1	0	0	0	1.0	0	0	0	.01
2	0	1.0	1.0	1.0	0	0	0	.01
3	0	0	0	1.0	0	.01	.01	.01

The second part of the maximum likelihood equation calculates $\frac{\partial L}{\partial \underline{\theta}}$ which is referred to as the gradient, and the third part of the program calculates $\frac{\partial^2 L}{\partial \underline{\theta}^2}$ known as the Fisher Information Matrix. The inverse of the Fisher Information Matrix is defined as the Cramer-Rao Lower Bound on the covariance of the θ estimates. The maximum likelihood approaches the lower bound asymptotically (8). The gradient and the Fisher Information Matrix are used to calculate the estimate of the value of the parameter to be identified, according to the equation

$$\underline{\theta}^* = \underline{\theta}_o - \Delta\theta \qquad (3.4)$$

where $$\Delta\theta = M^{-1}[\frac{\partial L}{\partial \underline{\theta}}]^T \quad \text{and} \quad M = \frac{\partial^2 L}{\partial \underline{\theta}^2}$$

As stated in Chapter 1, the maximum likelihood technique involves iterating through equations for L, $\frac{\partial L}{\partial \underline{\theta}}$, $\frac{\partial^2 L}{\partial \underline{\theta}^2}$, and $\underline{\theta}^*$ until $\Delta\underline{\theta}$ becomes smaller than some error criterion (ε). According to maximum likelihood theory as $\varepsilon \to 0$, $\underline{\theta}^*$ approaches the true identity of the unknown parameter. At the true value for $\underline{\theta}^*$ the likelihood function has its maximum, and according to maximum likelihood theory, for the technique to converge the likelihood function must have a maximum. Because it was not certain that the likelihood function for all the test cases had a maximum, the maximum likelihood processor was not allowed to converge. Instead the likelihood function was given specific values of theta for which to calculate L, ξ_{bias}, $\frac{\partial L}{\partial \underline{\theta}}$ and $\frac{\partial^2 L}{\partial \underline{\theta}^2}$. Theta ranged from .50 to 1.45 in increments of .05. In all of the simulated data generated, the following value for the unknown parameter was used:

$$\beta = 1.00$$

therefore the peak in the likelihood function for all cases should be at

$$\underline{\theta} = \beta = 1.00$$

For each value of theta in the range specified above, the maximum likelihood program iterated through its equations once calculating:

(1) L

(2) ξ_{bias}

(3) $\xi_{observation}$

(4) $\frac{\partial L}{\partial \underline{\theta}}$

(5) $\frac{\partial^2 L}{\partial \underline{\theta}^2}$

(6) $\Delta\theta$

(7) $\underline{\theta}*$

Figure 3.2 is a sample of the computer program output for one iteration. After the maximum likelihood program finished the iteration for $\theta = 1.45$ the five quantities listed above are plotted for the range of theta.

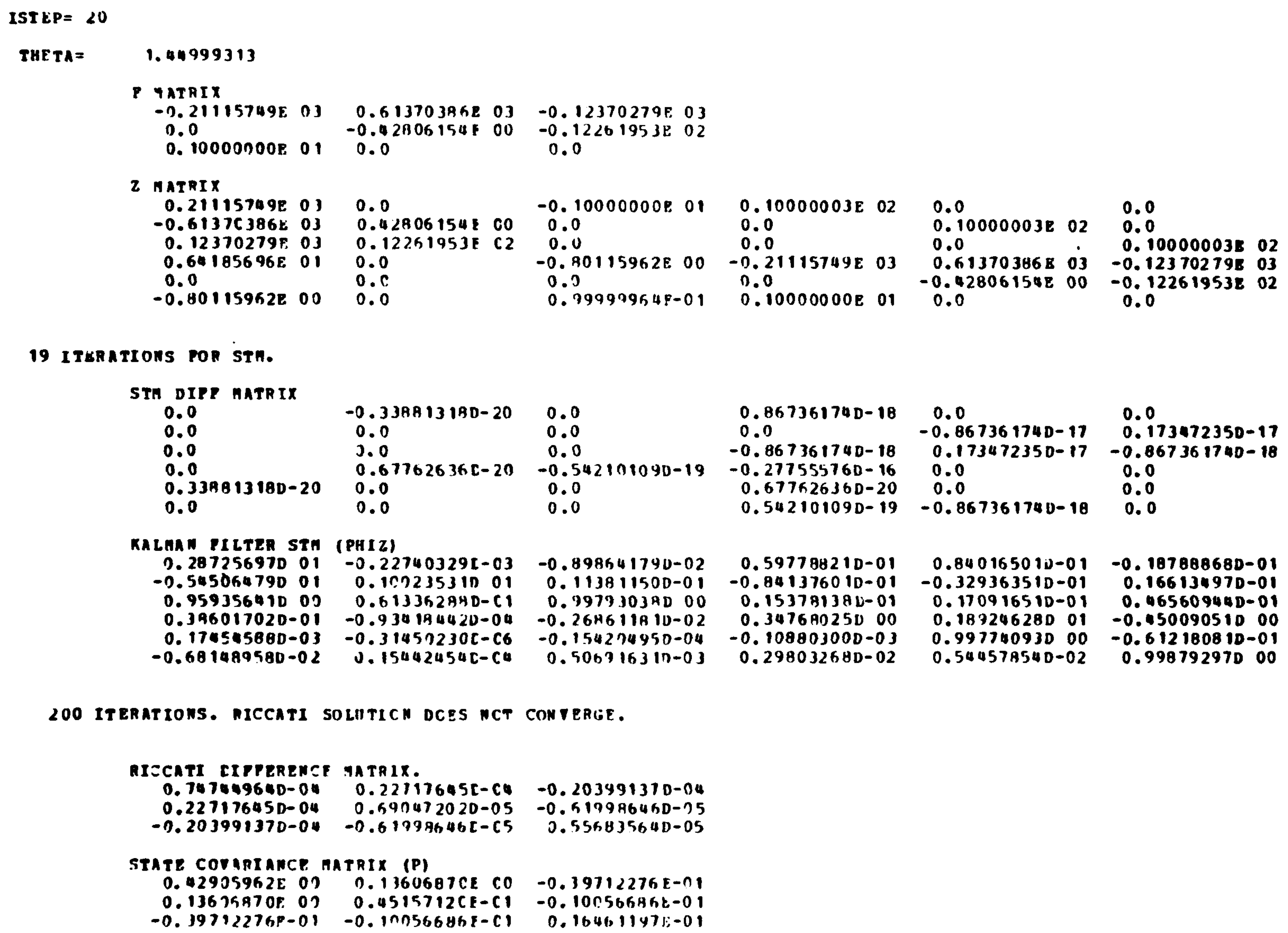
3.3 Test Case Results

A summary of the results of the maximum likelihood program for test cases 1 through 9 are given in Table 3.3. These results will be discussed in three sections. The first section will discuss the likelihood function for each test case. The second section will discuss the results for the gradient and the Fisher Information Matrix and the third will examine the application of the test data results to the analysis of real wheelset data.

3.3.1 Likelihood Function

In this section the test cases will be discussed in three groups as indicated on the right hand side of Table 3.1.

Examining test cases 8 and 9 first, Table 3.1 shows that a deterministic track input as well as a random track input was used to generate the simulated



```
ISTEP= 20

 THETA=      1.44999313

          F MATRIX
            -0.21115749E 03   0.61370386E 03  -0.12370279E 03
             0.0             -0.42806154E 00  -0.12261953E 02
             0.10000000E 01   0.0              0.0

          Z MATRIX
             0.21115749E 03   0.0             -0.10000000E 01   0.10000003E 02   0.0              0.0
            -0.61370386E 03   0.42806154E 00   0.0              0.0              0.10000003E 02   0.0
             0.12370279E 03   0.12261953E 02   0.0              0.0              0.0              0.10000003E 02
             0.64185696E 01   0.0             -0.80115962E 00  -0.21115749E 03   0.61370386E 03  -0.12370279E 03
             0.0              0.0              0.0              0.0             -0.42806154E 00  -0.12261953E 02
            -0.80115962E 00   0.0              0.99999964E-01   0.10000000E 01   0.0              0.0

  19 ITERATIONS FOR STM.

          STM DIFF MATRIX
             0.0             -0.33881318D-20   0.0              0.86736174D-18   0.0              0.0
             0.0              0.0              0.0              0.0             -0.86736174D-17   0.17347235D-17
             0.0              0.0              0.0             -0.86736174D-18   0.17347235D-17  -0.86736174D-18
             0.0              0.67762636D-20  -0.54210109D-19  -0.27755576D-16   0.0              0.0
             0.33881318D-20   0.0              0.0              0.67762636D-20   0.0              0.0
             0.0              0.0              0.0              0.54210109D-19  -0.86736174D-18   0.0

          KALMAN FILTER STM (PHIZ)
             0.28725697D 01  -0.22740329D-03  -0.89864179D-02   0.59778821D-01   0.84016501D-01  -0.18788868D-01
            -0.54506479D 01   0.10023531D 01   0.11381150D-01  -0.84137601D-01  -0.32936351D-01   0.16613497D-01
             0.95935641D 00   0.61336288D-01   0.99793038D 00   0.15378138D-01   0.17091651D-01   0.46560944D-01
             0.38601702D-01  -0.93418442D-04  -0.26861181D-02   0.34768025D 00   0.18924628D 01  -0.45009051D 00
             0.17454588D-03  -0.31450230D-06  -0.15420495D-04  -0.10880300D-03   0.99774093D 00  -0.61218081D-01
            -0.68148958D-02   0.15442454D-04   0.50691631D-03   0.29803268D-02   0.54457854D-02   0.99879297D 00

   200 ITERATIONS. RICCATI SOLUTION DOES NOT CONVERGE.

          RICCATI DIFFERENCE MATRIX.
             0.74744964D-04   0.22717645D-04  -0.20399137D-04
             0.22717645D-04   0.69047202D-05  -0.61998646D-05
            -0.20399137D-04  -0.61998646D-05   0.55683564D-05

          STATE COVARIANCE MATRIX (P)
             0.42905962E 00   0.13606870E 00  -0.39712276E-01
             0.13606870E 00   0.45157120E-01  -0.10056686E-01
            -0.39712276E-01  -0.10056686E-01   0.16461197E-01
```

Figure 3.2 Computer output for the last iteration of test case number 8.

Figure 3.2 (continued)

```
        KALMAN GAIN MATRIX (RKGM)
           0.42905970E 01   0.13606873E 01  -0.39712286E 00
           0.13606873E 01   0.45157129E 00  -0.10056686E 00
          -0.39712286E 01  -0.10056686E 00   0.16461200E 00


19 ITERATIONS FOR STM.

        STM DIFF MATRIX
          -0.27755576D-16   0.0              0.0
           0.67762636D-20   0.0              0.0
           0.54210109D-19  -0.86736174D-18   0.0

        SYSTEM STATE TRANSITION MATRIX (PHIX)
           0.34703362E 00   0.18917809E 01  -0.44986582E 00
          -0.11090885E-01   0.99773932E 00  -0.61217483E-01
           0.30864351E-02   0.55509247E-02   0.99875748E 00


        KALMAN FILTER SYSTEM MATRIX (FKF)
          -0.21544807E 03   0.61234302E 03  -0.12330566E 03
          -0.13606873E 01  -0.87963283E 00  -0.12161386E 02
           0.13971224E 01   0.10056686E 00  -0.16461200E 00


19 ITERATIONS FOR STM.

        STM DIFF MATRIX
          -0.27755576D-16   0.0              0.0
           0.0              0.0              0.0
           0.0             -0.86736174D-18   0.0


        STM FOR KALMAN FILTER SYSTEM (PHIK)
           0.33408942D 00   0.18615725D 01  -0.44243963D 00
          -0.42891837D-02   0.98796747D 00  -0.58943211D-01
           0.42553489D-02   0.81702378D-02   0.99744722D 00

        MEASUREMENT COVARIANCE MATRIX (E)
           0.52905959E 00   0.13606870E 00  -0.39712276E-01
           0.13606870E 00   0.14515704E 00  -0.10056686E-01
          -0.39712276E-01  -0.10056686E-01   0.11646116E 00


        NATURAL LOG OF THE DETERMINANT OF B= -0.50185900E 01

        VALUE OF THE LIKELIHOOD FUNCTION (RLIKE)
        RLIKE= 0.14254326E 04

        PARTIAL OF F WRT THETA (DFMTX)
          -0.14010136E 03   0.42124609E 03   0.0
           0.0              0.29521632E 00   0.0
           0.0              0.0              0.0


        Z MATRIX FOR SOLUTION TO PARTIAL(P)/PARTIAL(THETA)
           0.21544807E 01   0.13606873E 01  -0.13971224E 01   0.0              0.0              0.0
          -0.61234302E 03   0.87963283E 00  -0.10056686E 00   0.0              0.0              0.0
           0.12330566E 01   0.12161386E 02   0.16461200E 00   0.0              0.0              0.0
          -0.50425720E 01   0.89333475E-01   0.13072910E 01  -0.21544807E 03   0.61234302E 03  -0.12330566E 03
```

```
       0.89333475E-01   0.26662239E-01  -0.29688976E-02  -0.13606873E 01  -0.87963283E 00  -0.12161386E 02
       0.13072910E 01  -0.29688976E-02   0.0              0.13971224E 01   0.10056686E 00  -0.16461200E 00
&DIMEN
M=         6
&END

   19 ITERATIONS FOR STM.

       STM DIFF MATRIX
          0.0              0.0              0.0              0.0              0.0              0.0
          0.0              0.0              0.0              0.0              0.0              0.0
          0.0              0.0              0.0              0.0              0.0              0.0
          0.0             -0.54210109D-19   0.0             -0.27755576D-16   0.0              0.0
         -0.54210109D-19   0.0              0.13881118D-20   0.0              0.0              0.0
          0.0              0.0              0.0              0.0             -0.86736174D-18   0.0

       STM FOR SOLUTION TO PARTIAL(P)/PARTIAL(THETA)
          0.29103936D 01   0.11888656D-01  -0.12513818D-01   0.0              0.0              0.0
         -0.54918858D 01   0.98925088D 00   0.15326599D-01   0.0              0.0              0.0
          0.96643113D 00   0.63732332D-01   0.99791424D 00   0.0              0.0              0.0
         -0.30804079D-01   0.47269744D-03   0.40680290D-02   0.33408942D 00   0.18615725D 01  -0.44243963D 00
          0.29357480D-03   0.13265425D-03  -0.32378697D-04  -0.42891837D-02   0.98796747D 00  -0.58943211D-01
          0.11635022D-01   0.19156478D-04  -0.17233599D-04   0.42553489D-02   0.81702378D-02   0.99744722D 00

    200 ITERATIONS. RICCATI SOLUTION DOES NOT CONVERGE.

       RICCATI DIFFERENCE MATRIX.
          0.63945955D-05   0.20625284D-05  -0.91290541D-06
          0.20625284D-05   0.66859567D-06  -0.27106848D-06
         -0.91290541D-06  -0.27106848D-06   0.29388600D-06

       PARTIAL OF P WRT THETA (DELTAP)
          0.45339927E-01   0.20148747E-01   0.40192576E-03
          0.20148747E-01   0.67489929E-02  -0.16518242E-02
          0.40192576E-03  -0.16518242E-02   0.22276950E-02

       PARTIAL OF RKGM WRT THETA (DELTAK)
          0.45330936E 00   0.20148748E 00   0.40192567E-02
          0.20148748E 00   0.67489922E-01  -0.16518246E-01
          0.40192567E-02  -0.16518246E-01   0.22276953E-01

       GRADIENT - PARTIAL OF THE LIKELIHOOD FUNCTIONWRT THETA
       DELTAJ= 0.84175674E 02

       FISHER INFORMATION MATRIX
       DELJ2= 0.82025391E 02


       DTHETA=-0.10262146E 01

       THETAN= 0.42377853E 00
```

Figure 3.2 (continued)

Table 3.3

Summary of Results for Test Cases

Test Case	Likelihood Function Maximum θ =	Observation Term Maximum θ =	Bias Term Maximum θ =
1	.95	.95	N/A
2	N/A	.95	N/A
3	.95	.95	N/A
4	N/A	N/A	N/A
5	.95	.95	N/A
6	N/A	N/A	N/A
7	.65	N/A	N/A
8	.85	1.10	N/A
9	N/A	N/A	N/A

NOTE: N/A = not applicable, no maximum occurs or only a local maximum occurs.

wheelset data. As presented in Chapter 1, the wheelset equations of motion in state vector form are:

$$\dot{\underline{x}} = F\underline{x} + G\underline{w} \qquad (3.5)$$

where $\underline{w}$ = random track input

The Kalman Filter state estimates for this case are given by the equation

$$\hat{\underline{x}} = (F-KH)\hat{\underline{x}} + K\underline{z} \qquad (3.6)$$

o incorporate deterministic and random track inputs the following changes in the above two equations are necessary.

$$\dot{\underline{x}} = F\underline{x} + G\underline{u} + G\underline{w} \qquad (3.7)$$

where $\underline{u}$ = deterministic track input

$\underline{w}$ = random track input due to the uncertainty in $\underline{u}$

$$\dot{\hat{\underline{x}}} = (F-KH)\hat{\underline{x}} + K\underline{z} + G\underline{u} \qquad (3.8)$$

Although $\underline{u}$ is a white noise input it is called deterministic because it was an input into the system model which generated the data (equation 3.7) and it was an input into the Kalman Filter of the maximum likelihood program (equation 3.8). $\underline{w}$ is called a random track input because it appears as an input into the wheelset system (equation 3.7) but it is not an input into the Kalman Filter (equation 3.8). The Kalman Filter in the maximum likelihood program has the Q matrix as its only source of information about $\underline{w}$.

The difference between test cases 8 and 9 is the size of the random track input as compared to the deterministic track input. Case 9 has a larger random track input than case 8, which signifies that case 9 has more uncertainty in the deterministic track input than case 8. As Table 3.3 shows, for case 8 the likelihood function had a maximum but for case 9 the likelihood function did not have a maximum. This would suggest that the maximum likelihood method needs a well-defined deterministic track input if it is to identify a peak in the likelihood function. However, maximum likelihood theory has no restrictions.

This suggests that the maximum likelihood processor may need more measurements--a longer data record--when the deterministic track input has a large amount of uncertainty. In reference (6) an increase in the number of observations is shown to cause a more pronounced maximum in the likelihood function. Although there are no quantitative results which prove more measurements will produce a peak in the likelihood function for the dynamically scaled wheelset case, this is an area in which further research should be done.

For all of the test cases in group 1, the Kalman Filter in the maximum likelihood program was given the following values for Q and R, as shown in Table 3.1 and Table 3.2.

$$Q = 1.0$$

$$R = \begin{bmatrix} .01 & 0 & 0 \\ 0 & .01 & 0 \\ 0 & 0 & .01 \end{bmatrix}$$

These five test cases were run for diagnostic purposes but have been included here because they exhibit some interesting trends.

Case 2 is called the perfect observation case in the literature. In reference (7) the likelihood function is shown to reduce to equation 3.9 for this case.

$$L = \sum_{i=1}^{N} {}^{T}(t_i)Q^{-1}\,(t_i) \qquad (3.9)$$

where $$\nu(t_i) = z(t_i) - \Phi z(t_{i-1}) \qquad (3.10)$$

when Q is known. Essentially equation 3.9 says that the likelihood function simplifies to the observation term for the perfect measurement case. The maximum likelihood processor did not analyze this limiting case because equations 3.9 and 3.10 were not incorporated into the maximum likelihood algorithm for test case number 2. The results of test case 2 do provide insight into the performance of maximum likelihood processor when it has incorrect or imprecise information about $\underline{w}$ or $\underline{v}$. Figures 3.3, 3.4 and 3.5 contain plots of the output of the maximum likelihood program for test case 2.

Test case 2 has a random track input only which makes it a limiting case for the effect of $\underline{u}$ versus $\underline{w}$ that was examined in cases 8 and 9. Test case 2 suggests that the more randomness there is in the track input, the more difficulty the maximum likelihood processor has in identifying maximum for the likelihood function. There are no theoretical limitations of this type for maximum likelihood. As in test cases 8 and 9 one practical aspect of implementing the maximum likelihood technique that should be investigated is data record length. Although several assumptions were made in the generation of the simulated data as discussed in section 1.4 it is believed that these assumptions will not seriously affect the performance of the maximum likelihood processor.

Case number 3 is a special case of maximum likelihood identification because there is no random track input. The likelihood function reduces to the observation term for this case also;however;the weighting matrix is different than in the no measurement noise case.

$$L = \sum_{i=1}^{N} \nu^T(t_i)R^{-1}\nu(t_i) \qquad (3.11)(\underline{7})$$

where

$$\nu(t_i) = z(t_i) - H\hat{x}(t_i) \qquad (3.12)(\underline{7})$$

when R is known. In this limiting case a Kalman Filter is used by the maximum likelihood processor as shown in equation 3.12, however the weighting matrix is not dependent on P but only on R. The observation term calculated by the maximum likelihood program for test case 3 was

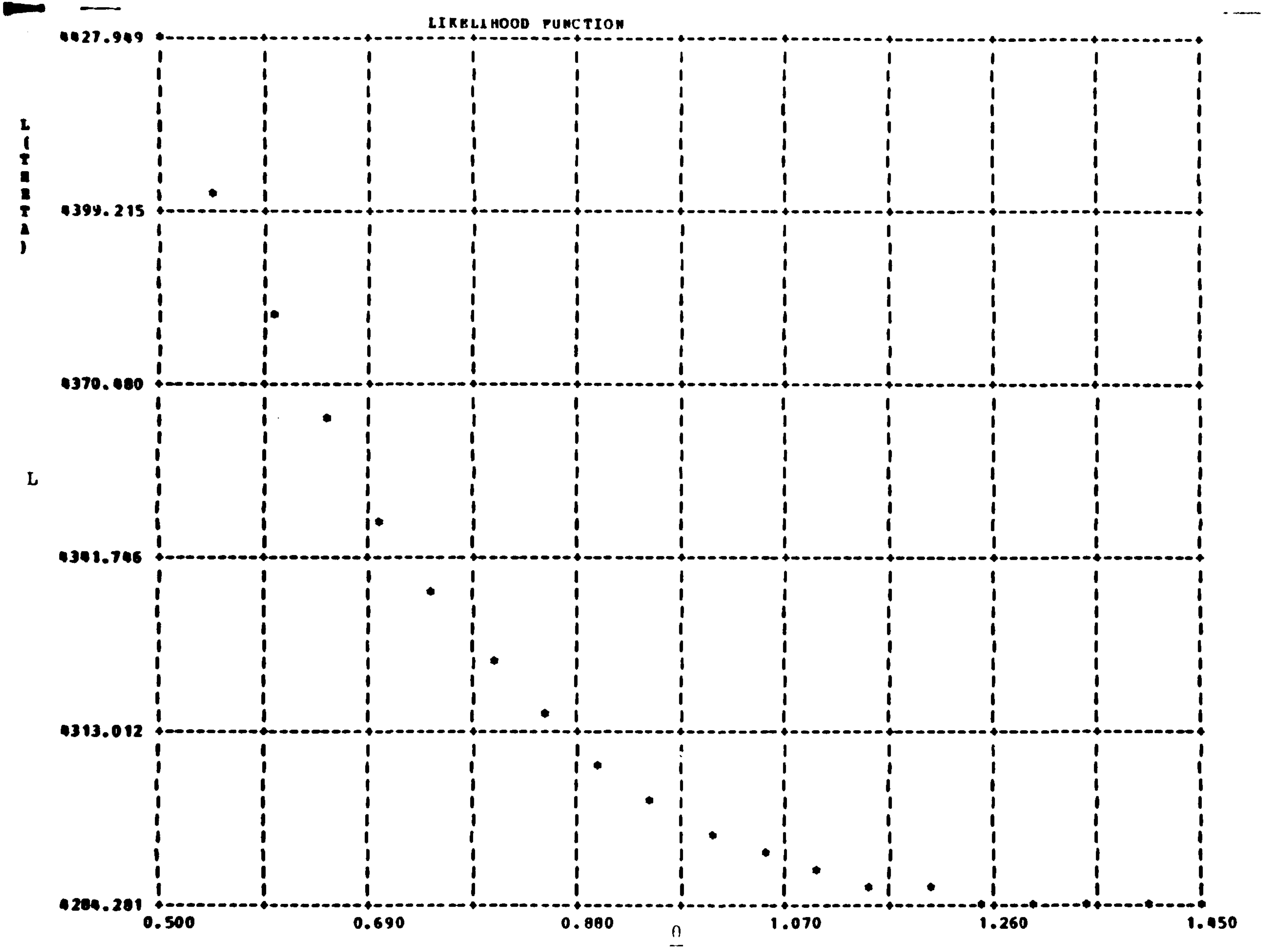


Figure 3.3 Plot of the likelihood function for test case 2.

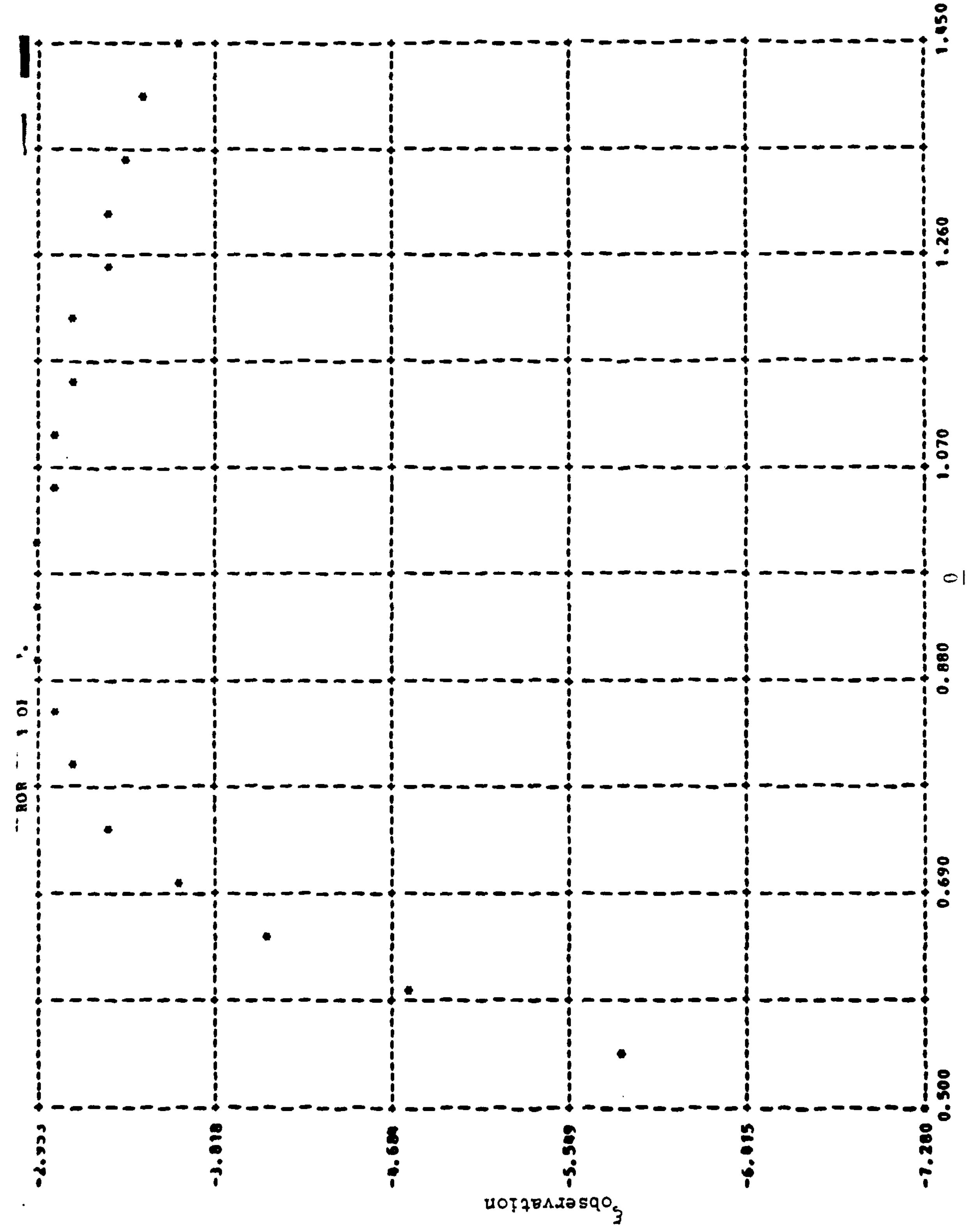


Figure 3.4 Plot of the observation term for test case 2.

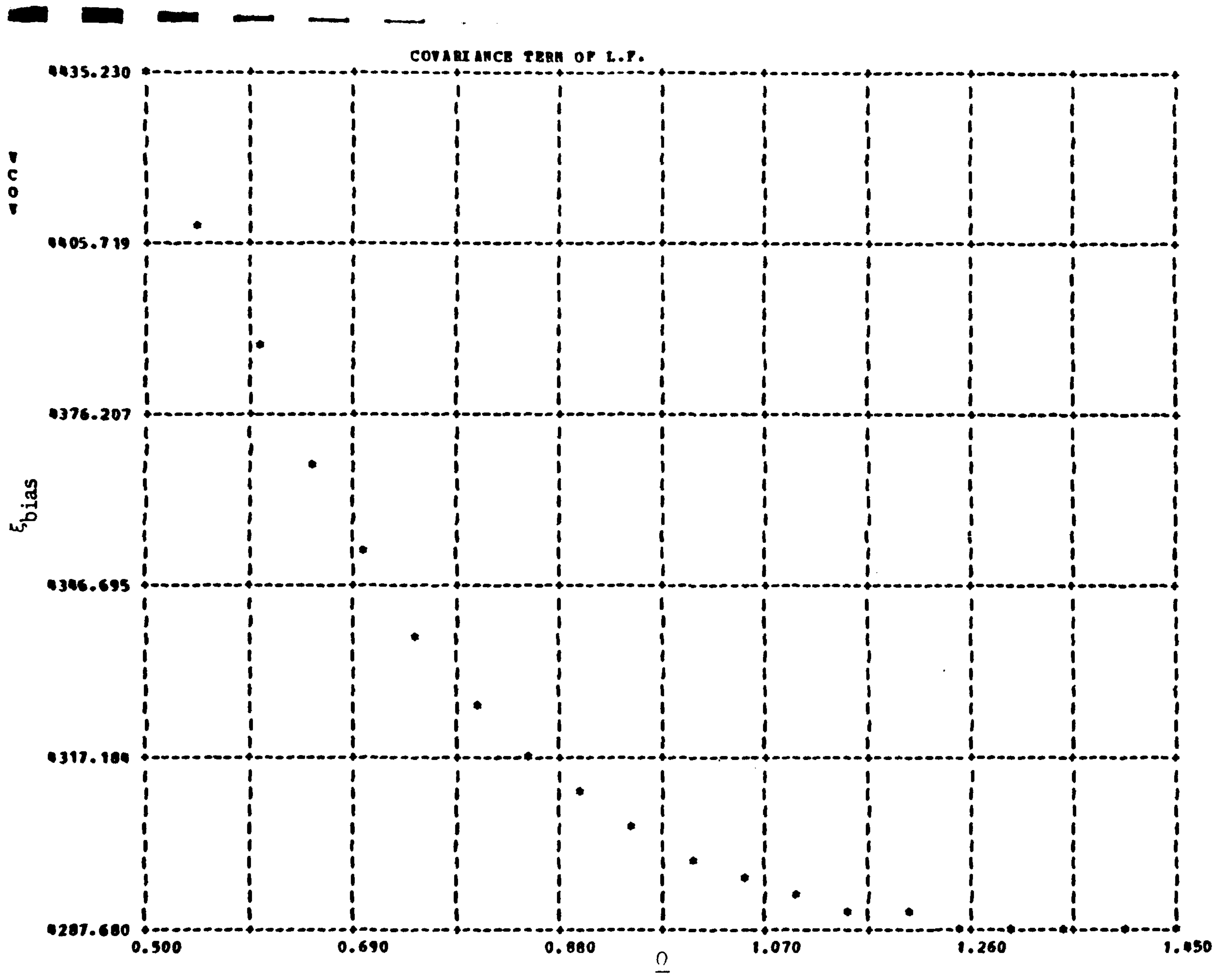


Figure 3.5 Plot of the bias term for t st case 2.

$$\xi = -\frac{1}{2}\sum_{i=1}^{N} \nu^T(t_i)B^{-1}\nu(t_i) \tag{3.13}$$

where
$$B = HPH^T + R \tag{3.14}$$

and
$$\nu(t_i) = z(t_i) - H\hat{x}(t_i) \tag{3.15}$$

The state estimates as calculated by the maximum likelihood program are not as accurate as they could be because the Kalman Filter was given an incorrect value for Q (Table 3.2). Also the weighting matrix used in equation 3.13 is very different from the weighting matrix used in equation 3.11. Because the maximum likelihood algorithm for test case 3 did not incorporate equation 3.11 and because Q was incorrect, the limiting case of no process noise was not analyzed correctly.

Test case 1 is a combination of the two limiting cases presented above. The likelihood function reduces to:

$$L = \sum_{i=1}^{N} \nu^T(t_i)\nu(t_i) \tag{3.16}$$

where
$$\nu(t_i) = z(t_i) - \Phi z(t_{i-1}) \tag{3.17}$$

As equation 3.17 shows a Kalman Filter would not be used in this case because there is no random track input and no measurement noise. As for the other test cases in group 1 the values for Q and R used by the maximum likelihood program for test case 1 were:

$$Q = 1.0$$

$$R = \begin{bmatrix} .01 & 0 & 0 \\ 0 & .01 & 0 \\ 0 & 0 & .01 \end{bmatrix}$$

Because of this discrepancy, the results obtained from test case 1 are not necessarily those that would be obtained if the no random track input-no measurement noise case were implemented correctly using equations 3.16 and 3.17.

Test case 4 and test case 5 are the only members of group 1 for which the maximum likelihood program was given correct values for Q and R. The results of test cases 4 and 5 given. As demonstrated in other test cases the maximum likelihood processor did not identify a peak in the likelihood function when the only input driving the wheelset system is a random track input. From the test cases performed as part of this research program no conclusive explanations can be given for the above problem. As suggested before, a longer data record in these cases may be necessary however there is no direct evidence to support this conclusion.

Test cases 6 and 7 were run to see if the ratio of Q to R had any affect on the performance of the maximum likelihood processor. For test case 6 the ratio of Q to R is the same as that for test cases 4 and 5. According to maximum likelihood if the values of Q and R are changed but their ratio remains constant then the observation term will remain the same but the bias term will change. Graphs of the observation term for test case 4 and test case 6 are presented in Figure 3.6 and 3.7 respectively. Comparing these two plots shows that the observation term did not change between these two cases. Comparing Figures 3.8 and 3.9 it can be seen that the bias term did change its range of values, but not its general shape. The likelihood functions for case 4 and case 6 are given in Figures 3.10 and 3.11. The likelihood function has the same general shape for each case, however, the location of the local maximum does change. This agreement between test cases 4 and 6 further supports the validity of the maximum likelihood algorithm given in Chapter 1, and the computer program which implements it.

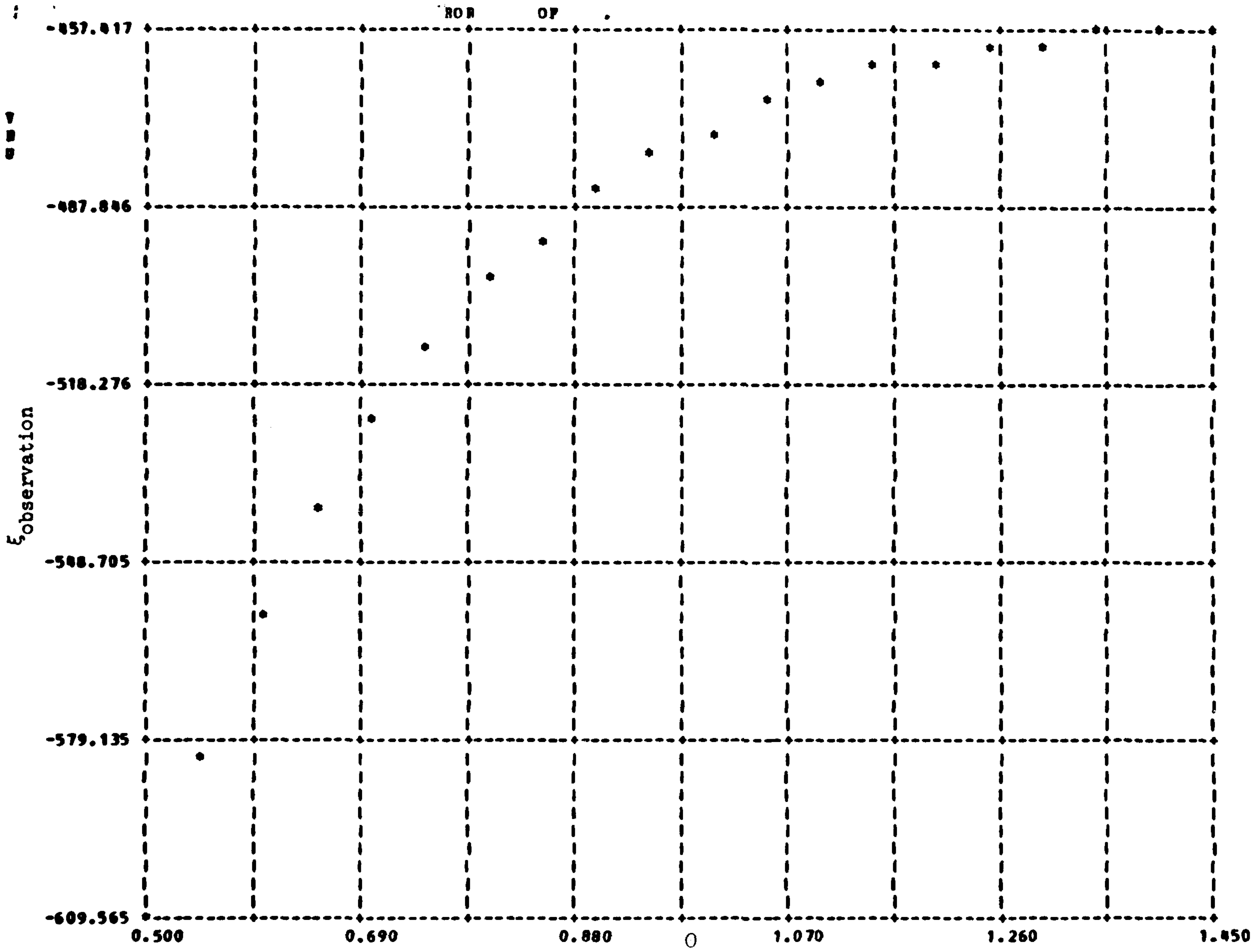


Figure 3.6 Plot of the observation term for test case 4.

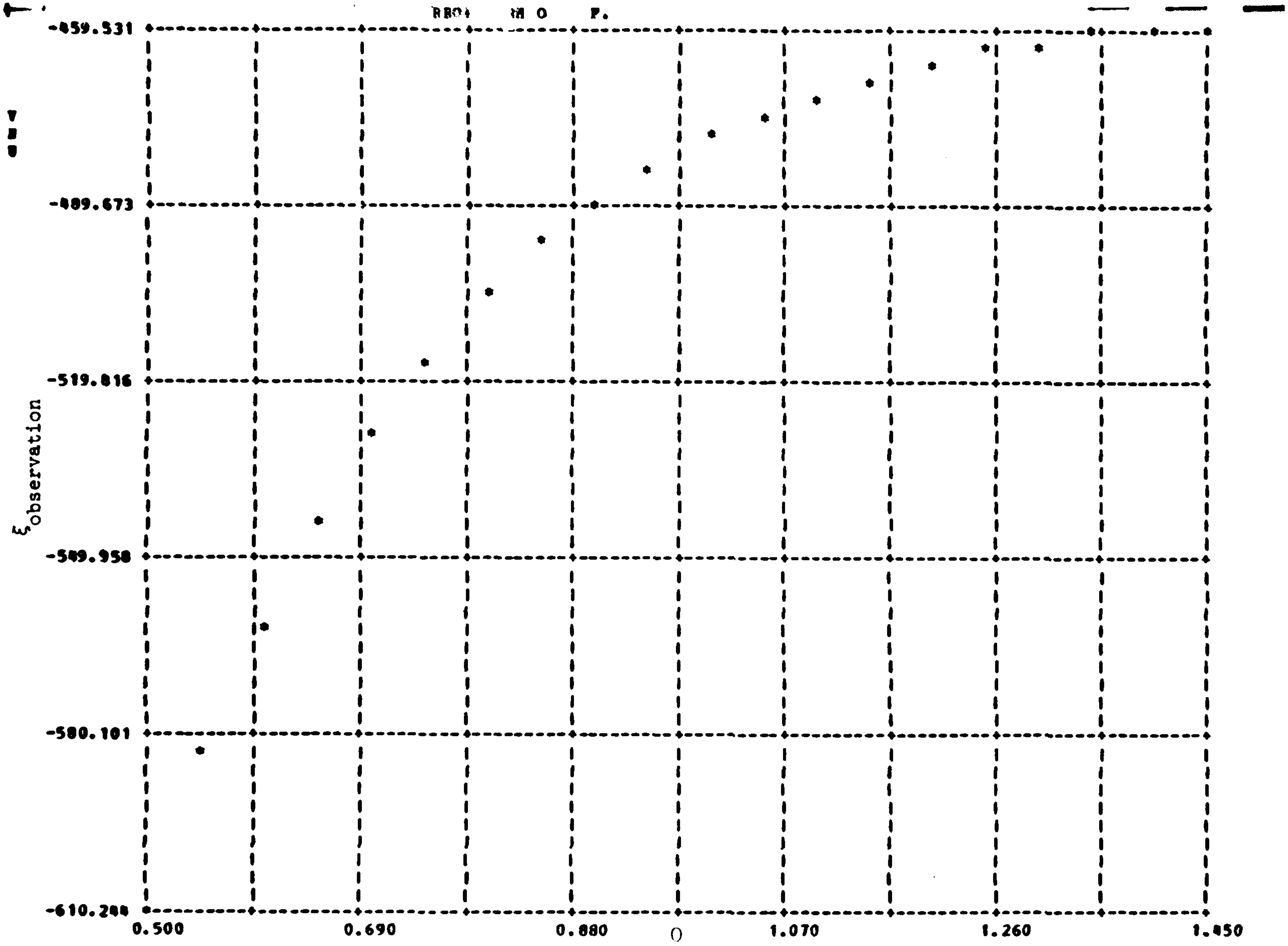


Figure 3.7 Plot of the observation term for test case 6.

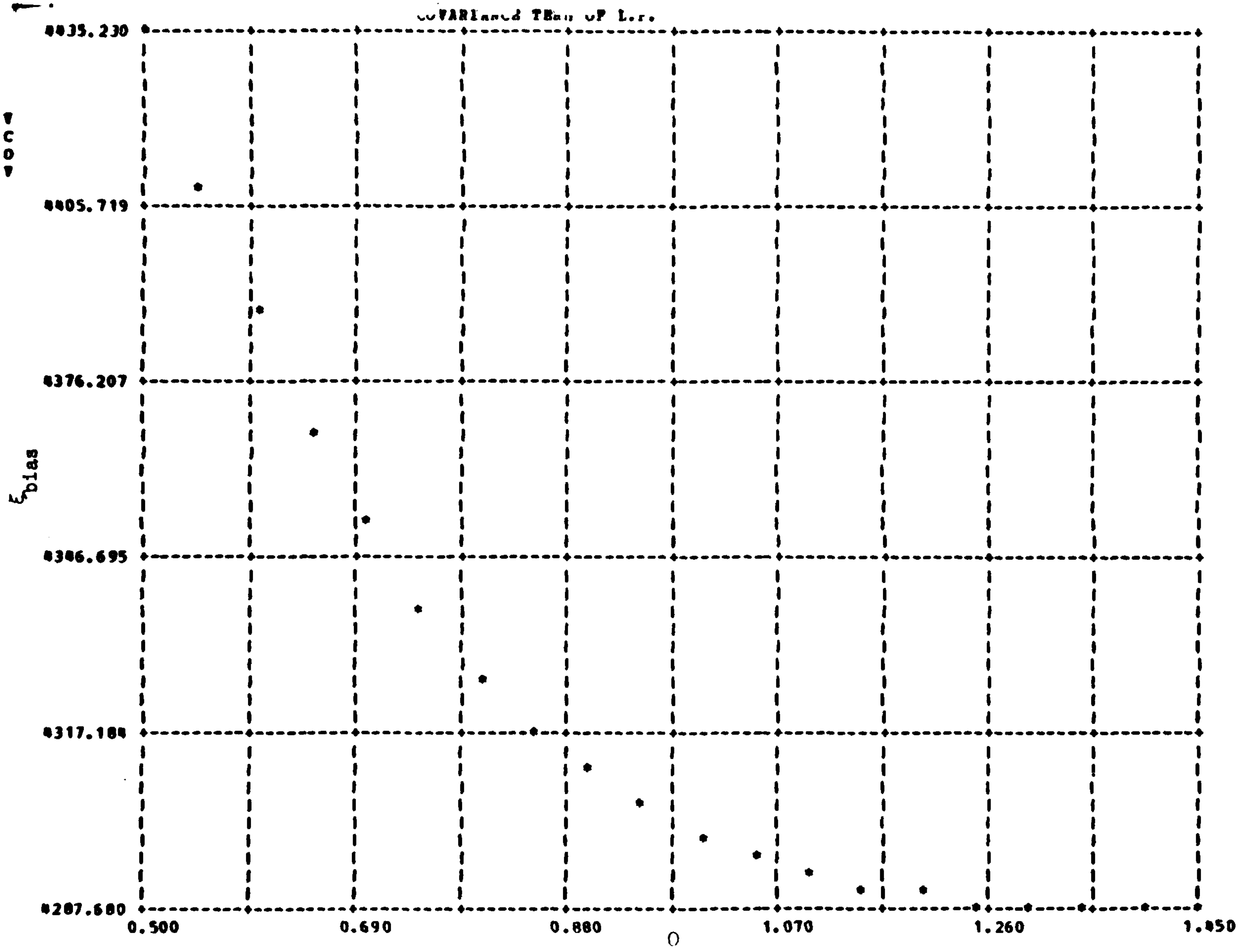


Figure 3.8 Plot of the bias term for test case 4.

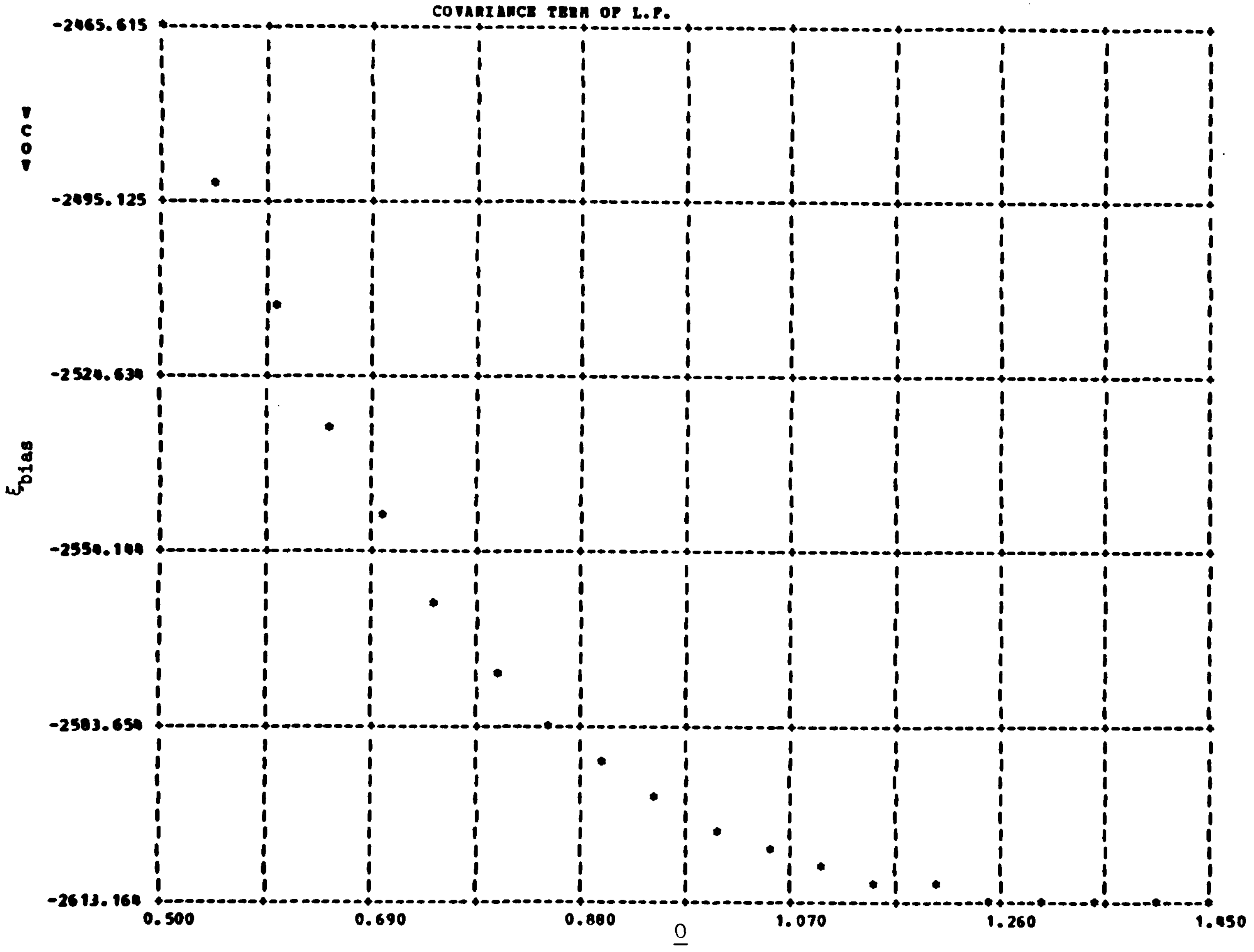


Figure 3.9 Plot of the bias term for test case 6.

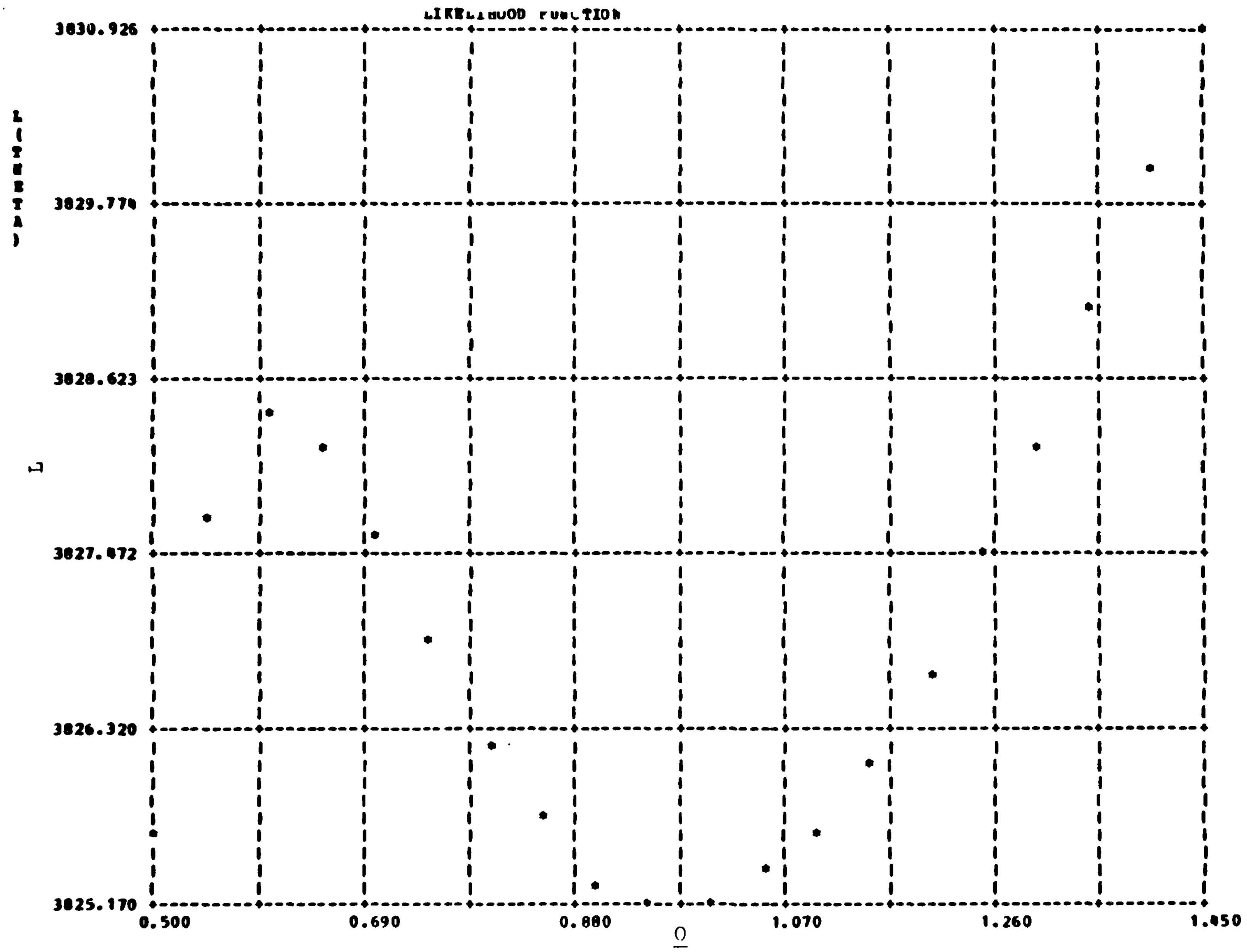


Figure 3.10 Plot of the likelihood function for test case 4.

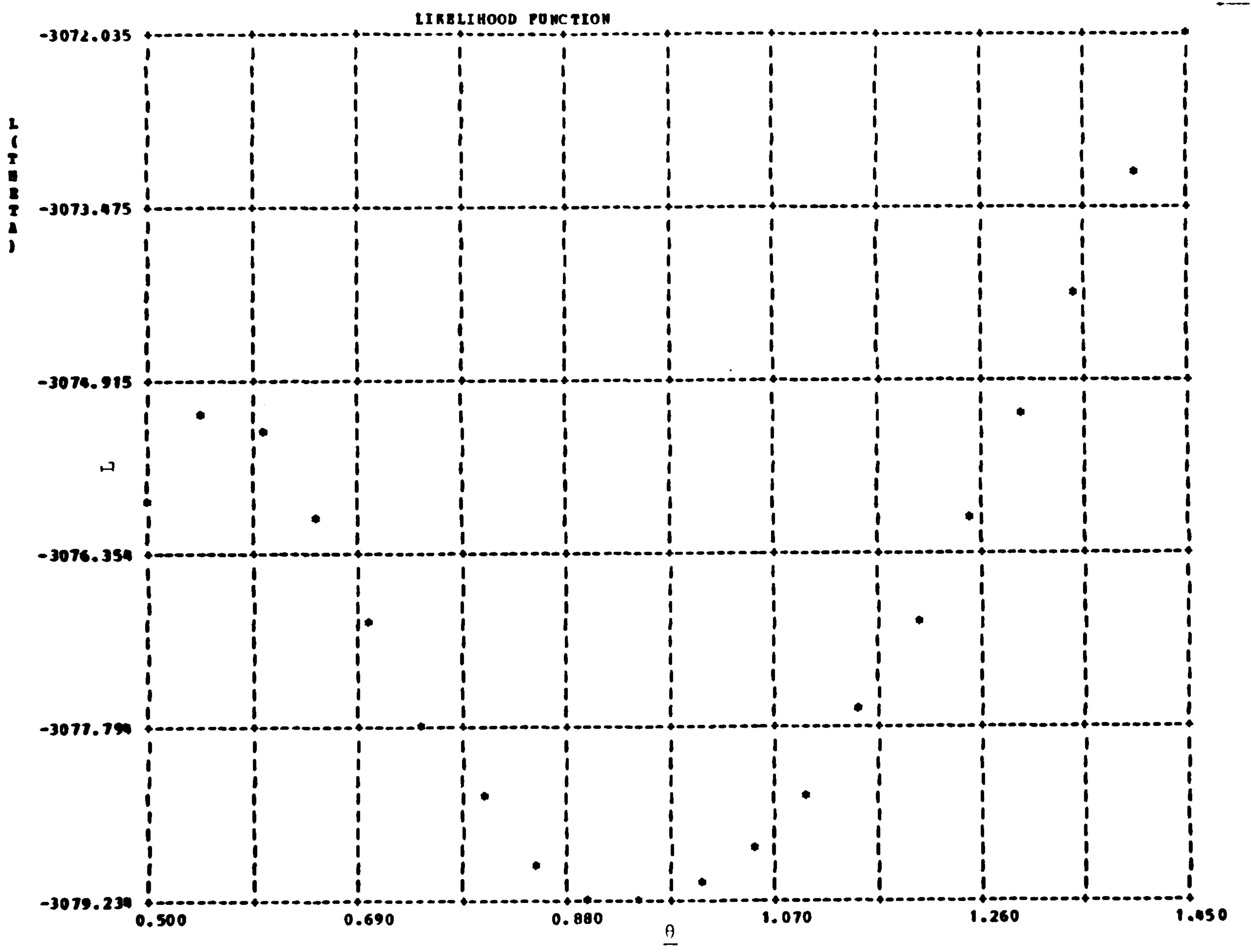


Figure 3.11 Plot of the likelihood function for test case 6.

Test case 6 has as its only input a random input disturbance and in accordance with the results of other similar test cases, its likelihood function does not have a maximum. Test case 7 also has a random disturbance as the only input to the wheelset model; however, the likelihood function in the case 7 results does have a maximum at $\theta = .65$. A plot of this likelihood function is given in Figure 3.12. The results of test case 7 indicate that the test data record was long enough for the maximum likelihood processor to identify a peak in the likelihood function, although the peak occurred at $\underline{\theta} = .65$ instead of $\underline{\theta} = 1.00$. There is no indication that a longer record will shift the peak from $\theta = .65$ to $\theta = 1.00$, although this is an area which needs further research. The fact that the measurement noise in case 7 is much smaller than in case 6 may be the reason the maximum likelihood processor was able to identify a peak in the likelihood function.

There is another possible explanation for the problems the likelihood function had in identifying a peak in the likelihood function. As discussed in section 1.4 a $\Delta t = .005$ sec was used to generate the test data. This time step resulted in 6 data points per cycle of the highest frequency response of the wheelset. The maximum likelihood processor may need more information about this particular mode of response of the wheelset. This could be accomplished by using a smaller Δt. However, because the present method used to generate the simulated data is restricted to 1000 data points per state variable, a smaller Δt would produce data with fewer cycles of the low frequency response of the wheelset. In order to investigate the effects of a smaller Δt a new method for generating the simulated data will have to be developed.

Since all of the simulated wheelset data was generated with the same Δt, and for several cases a peak in the likelihood function was identified there

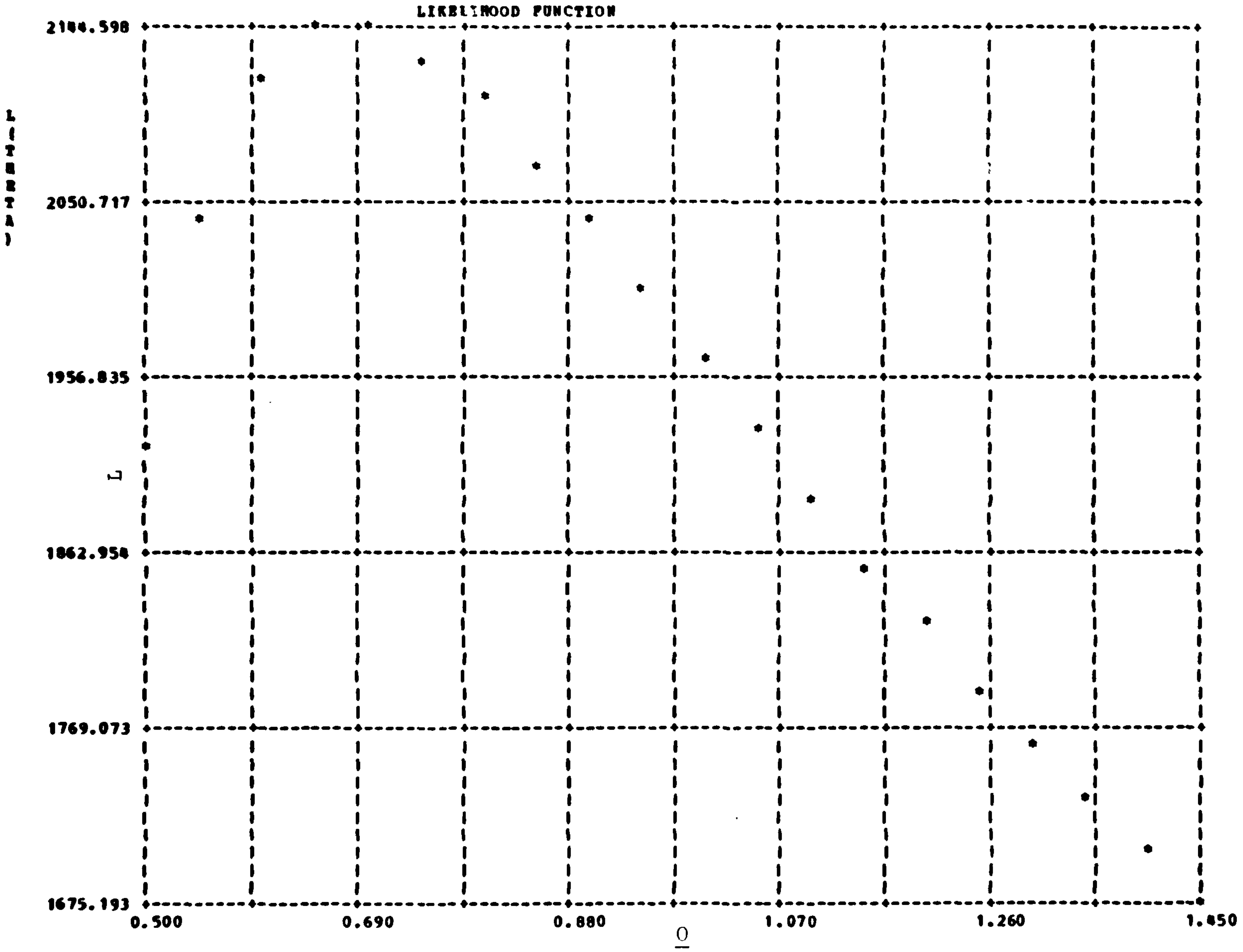


Figure 3.12 Plot of the likelihood function for test case 7.

does not seem to be a need for a smaller Δt. However, in those cases where the simulated data was generated using a random track input, the maximum likelihood processor was given minimal information about the input into the wheelset model that generated the data--only the covariance matrix for the random track input. To make up for this limited information about the input the maximum likelihood processor may need better information in other areas. This "better" information includes more data points per cycle of the high frequency response mode of the wheelset--obtained by a smaller Δt--and more observations of the low frequency response of the wheelset--obtained with a larger data record.

3.3.2 Gradient and Fisher Information Matrix

The task of evaluating the performance of the maximum likelihood processor in calculating $\frac{\partial L}{\partial \underline{\theta}}$ and $\frac{\partial^2 L}{\partial \underline{\theta}^2}$ is considerably more difficult than evaluating the calculation of L itself. There are two methods that can be used to check the gradient and second partial. The first method involves curve fitting likelihood function and gradient data points with n^{th} order polynomials. The derivatives can then be calculated analytically using the polynomial equation. The problem experienced with this method is that the data points do not represent a smooth function and inflection points which do not exist in the data are created. These inflection points cause large errors in the calculation of the derivative. The second method, and the one used to analyze the test data results, is explained in Figure 3.13. From Figure 3.13

$$m_{32} = \frac{y_3 - y_2}{x_3 - x_2} \tag{3.18}$$

$$m_{43} = \frac{y_4 - y_3}{x_4 - x_3} \tag{3.19}$$

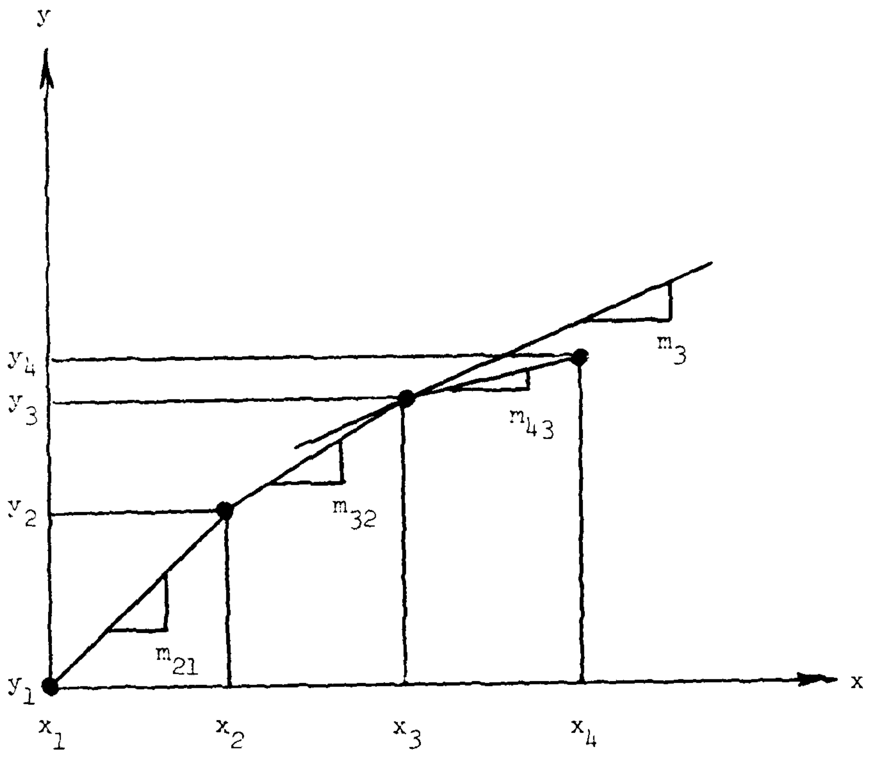


Figure 3.13 Average slope method for checking the gradient and second partial of the likelihood function.

$$m_3 = \frac{m_{43} + m_{32}}{2} \qquad (3.20)$$

$$m_1 = m_{21} \qquad (3.21)$$

$$m_4 = m_{43} \qquad (3.22)$$

Equations 3.18 through 3.22 were used to calculate the derivatives of the likelihood function and the gradient. The slope at the first data point was set equal to the slope between the first two data points. The slope at the last data point was set equal to the slope between the last two data points. This method of averaging slopes works very well when the data points form a smooth function. In many cases the likelihood function data points and the gradient data points do not form smooth functions and the derivatives calculated using this second method can have large errors. Since the only objective was to check the reasonableness of the gradient and second partial data, the second method of averaging slopes was used. In general the difference between the gradient and the derivative of L as determined by the average slope method is on the order of 5%. The difference between the second partial and the slope of the gradient as determined by the averaging method was considerably larger--on the order of 35%. This difference for the second partial was considerably larger for test cases 4 and 6. The reason the difference between the second partial and the slope of the gradient may be so large for test cases 4 and 6 is that determining derivatives by averaging slopes produces large errors for data points which do not form smooth functions. The data points for the [illegible] cases 4 and 6 are not as smooth as in other cases. See Figures [illegible] large discrepancy is most probably due to errors in the [illegible] Information Matrix rather than in the calculation [illegible] the gradient and the second partial was

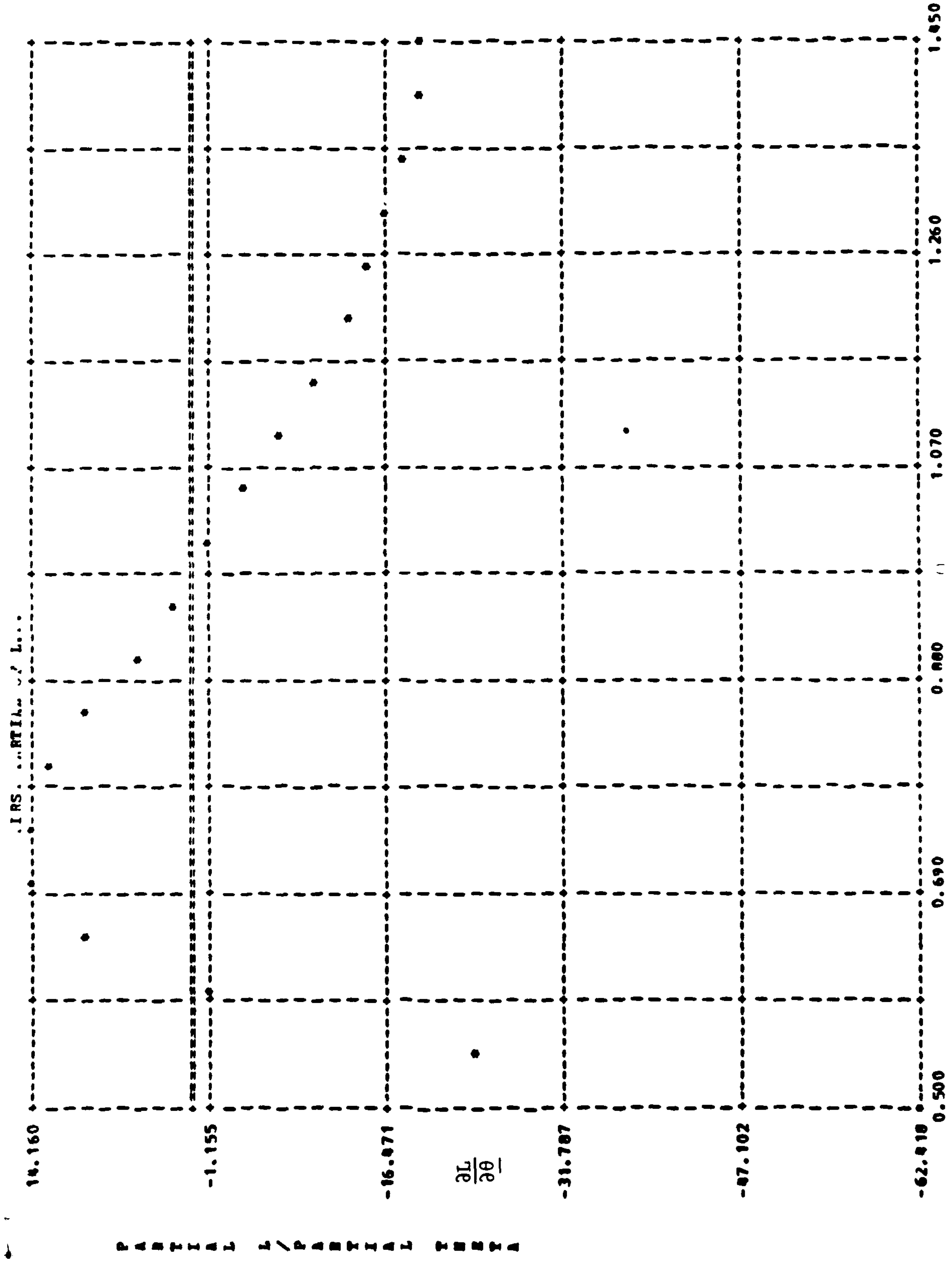


Figure 3.14 Plot of the gradient for test graph 4.

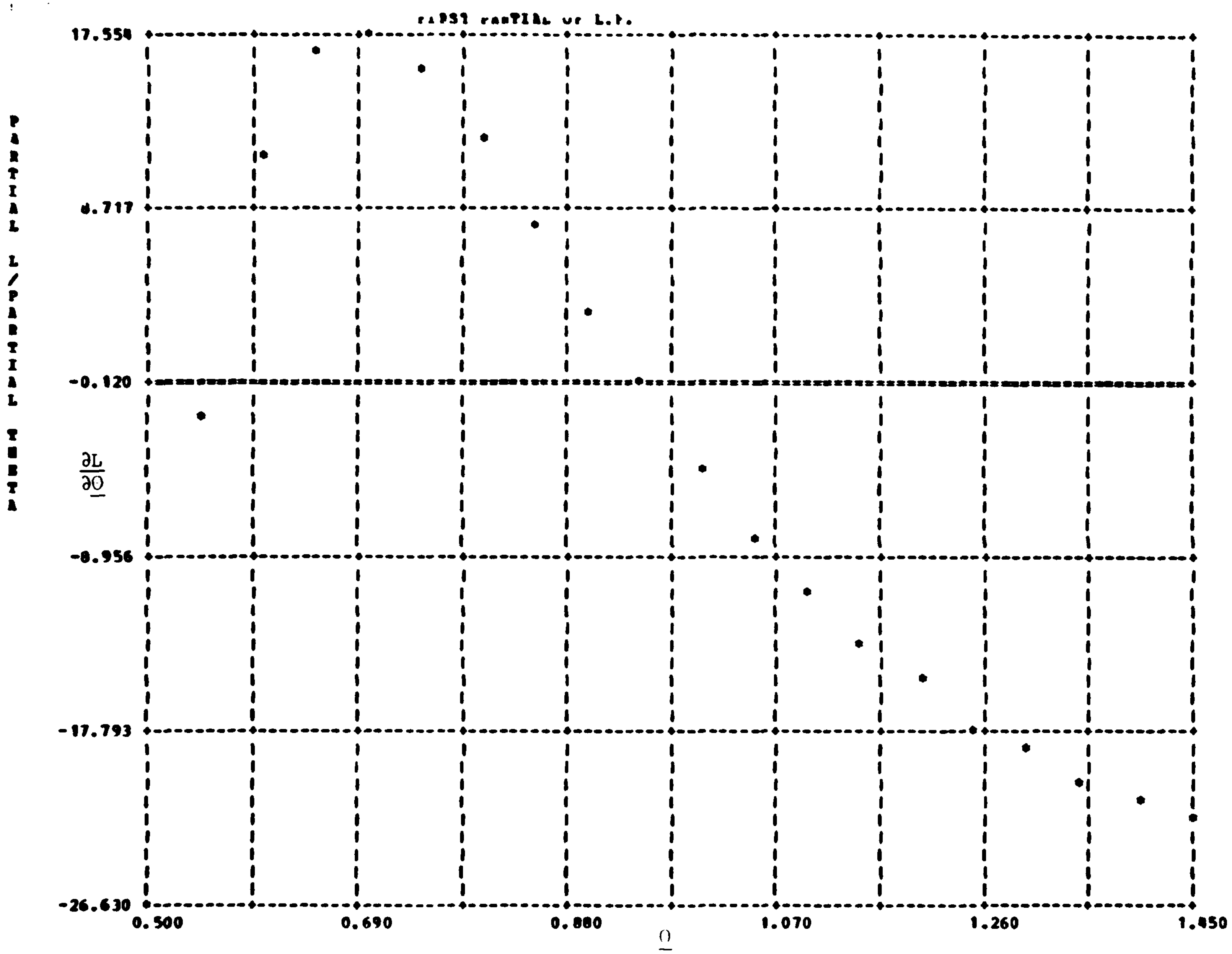


Figure 3.15 Plot of the gradient for test case 6.

not carried out for test cases 8 and 9 because based on the results for test cases 1 through 7 it was concluded that the quasilinearization section of the maximum likelihood algorithm was implemented correctly in the maximum likelihood computer program.

3.4 Application of Results to Actual Wheelset Data

The test data cases were analyzed by the maximum likelihood program for the following reasons:

(1) to ensure that the computer program was working properly.

(2) to identify any potential problems that may arise when the actual wheelset data is analyzed.

(3) to make recommendations for further research, based on the conclusions reached in (2) above.

The first objective was attained as documented in the previous sections. The purpose of this section is to discuss items (2) and (3).

In every case except one where the only input into the wheelset model was the random track input the likelihood function did not have a maximum. Several times in the previous section the point has been suggested that the likelihood function failed to have a maximum becuase the data record needed to be longer. The test cases run as part of this research program do not form a large enough experimental base to state the above point with any certainty. It is suggested that further research in this area be directed to find out how the length of the data record affects the performance of the maximum likelihood processor. As mentioned in Chapter 1, the present method of generating simulated data is limited to 1000 observations because of limitations in computer storage, therefore another method of generating simulated data will have to be designed.

Another area of research which should be examined is the effect of a smaller Δt in generating the test data. A smaller Δt would provide more information about the high frequency response mode of the wheelset. In order not to lose information about the low frequency mode, data records longer than 1000 points per state variable will have to be used. As pointed out above this requires a new method of generating simulated data.

Appendix H contains plots of the likelihood function, observation time and bias term for test cases 1 through 9.

CONCLUSION

As stated in the Introduction, the objective of this research program was the development and implementation of a maximum likelihood parameter identification algorithm applicable to a dynamically scaled wheelset model. The following is a summary of the conclusions that can be drawn from the results of this research program.

1) The maximum likelihood parameter identification equations can be tailored to the problem of identifying creep coefficients for the dynamically scaled wheelset model.

2) The reasonable results obtained for many of the test cases proves that the implementation of the maximum likelihood algorithm in Fortran IV was performed correctly.

3) More research involving simulated wheelset data is necessary before actual wheelset data can be processed. It is recommended that the following areas be investigated.

   a) the effect of data record length on the performance of the maximum likelihood processor

   b) advantages in representing the track input as a deterministic input with uncertainty as compared to representing the track input as a random track input

   c) the effect of a smaller Δt on the performance of the maximum likelihood processor.

The primary motivation for the above three recommendations is that it seems reasonable to assume the maximum likelihood processor works better when it has more information about the system and about the inputs into the system. If the track input is a random track input then the covariance matrix is the only information the maximum likelihood processor receives about this random track input. In order to make up for this limited information about the input into the wheelset system, the maximum likelihood processor should receive better information in other areas. A smaller Δt and a longer data record provide more information about the wheelset high frequency response mode and low frequency model respectively. This increase in the quantity of information could be considered better from the maximum likelihood processor point of view.

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Appendix A

DERIVATION OF $\frac{\partial\hat{\underline{x}}(t)}{\partial\underline{\theta}}$

$$\dot{\hat{\underline{x}}}(t) = F(t)\hat{\underline{x}}(t) + L(t)\underline{u}(t) + K(t)[\underline{z}(t) - H(t)\hat{\underline{x}}(t)] \tag{A.1}$$

$$\frac{\partial\dot{\hat{\underline{x}}}(t)}{\partial\underline{\theta}} = G(t)\,\frac{\partial\hat{\underline{x}}(t)}{\partial\underline{\theta}} + \frac{\partial F(t)}{\partial\underline{\theta}}\,\hat{\underline{x}}(t) + L(t)\,\frac{\partial\underline{u}(t)}{\partial\underline{\theta}} + \frac{\partial L(t)}{\partial\underline{\theta}}\,\underline{u}(t)$$

$$+ K(t)\,\frac{\partial\underline{z}(t)}{\partial\underline{\theta}} + \frac{\partial K(t)}{\partial\underline{\theta}}\,\underline{z}(t) - K(t)[H(t)\,\frac{\partial\hat{\underline{x}}(t)}{\partial\underline{\theta}} + \frac{\partial H(t)}{\partial\underline{\theta}}\,\hat{\underline{x}}(t)]$$

$$- \frac{\partial K(t)}{\partial\underline{\theta}}\,H(t)\hat{\underline{x}}(t) \tag{A.2}$$

Since the input disturbance is deterministic $\frac{\partial\underline{u}(t)}{\partial\underline{\theta}} = 0$. The measurements $\underline{z}$ are independent of θ therefore $\frac{\partial\underline{z}}{\partial\underline{\theta}} = 0$. Therefore equation A-3 reduces to:

$$\frac{\partial\hat{\underline{x}}(t)}{\partial\underline{\theta}} = F(t)\,\frac{\partial\hat{\underline{x}}(t)}{\partial\underline{\theta}} + \frac{\partial F(t)}{\partial\underline{\theta}}\,\hat{\underline{x}}(t) + \frac{\partial L(t)}{\partial\underline{\theta}}\,\underline{u}(t) + \frac{\partial K(t)}{\partial\underline{\theta}}\,\underline{z}(t)$$

$$- K(t)H(t)\,\frac{\partial\hat{\underline{x}}(t)}{\partial\underline{\theta}} - K(t)\,\frac{\partial H(t)}{\partial\underline{\theta}}\,\hat{\underline{x}}(t) - \frac{\partial K(t)}{\partial\underline{\theta}}\,H(t)\hat{\underline{x}}(t) \tag{A.3}$$

Appendix B

SOLUTION TO $\dfrac{\partial P(t_i)}{\partial \underline{\theta}}$

The matrix Riccati equation is

$$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) - P(t)H^T(t)R^{-1}(t)H(t)P(t) \qquad \text{(B.1)}$$

Taking the partial derivative of both sides with respect to $\underline{\theta}$:

$$\frac{\partial P(t)}{\partial \underline{\theta}} = F(t)\,\frac{\partial P(t)}{\partial \underline{\theta}} + \frac{\partial F(t)}{\partial \underline{\theta}}\,P$$

$$P(t)\,\frac{\partial (F^T(t))}{\partial \underline{\theta}} + \frac{\partial P(t)}{\partial \underline{\theta}}\,F^T(t)$$

$$+ G(t)Q(t)\,\frac{\partial (G^T(t))}{\partial \underline{\theta}} + G(t)\,\frac{\partial Q(t)}{\partial \underline{\theta}}\,G^T(t) + \frac{\partial G(t)}{\partial \underline{\theta}}\,Q(t)G^T(t)$$

$$- P(t)H^T(t)R^{-1}(t)H(t)\,\frac{\partial P(t)}{\partial \underline{\theta}} - P(t)H^T(t)R^{-1}(t)\,\frac{\partial H(t)}{\partial \underline{\theta}}\,P(t)$$

$$-P(t)H^T(t)\,\frac{\partial (R^{-1}(t))}{\partial \underline{\theta}}\,H(t)P(t) - P(t)\,\frac{\partial (H^T(t))}{\partial \underline{\theta}}\,R^{-1}(t)H(t)P(t)$$

$$- \frac{\partial P(t)}{\partial \underline{\theta}}\,H^T(t)R^{-1}(t)H(t)P(t) \qquad \text{(B.2)}$$

Post-multiply both sides of (B.2) by $\underline{y}$. The time notation will be dropped in further equations however all matrices are still time dependent.

$$\frac{\partial \dot{P}}{\partial \underline{\theta}}\,\underline{y} = F\,\frac{\partial P}{\partial \underline{\theta}}\,\underline{y} + \frac{\partial F}{\partial \underline{\theta}}\,P\underline{y} + P\,\frac{\partial F^T}{\partial \underline{\theta}}\,\underline{y} + \frac{\partial P}{\partial \underline{\theta}}\,F^T\underline{y} + GQ\,\frac{\partial G^T}{\partial \underline{\theta}}\,\underline{y} + G\,\frac{\partial Q}{\partial \underline{\theta}}\,G^T\underline{y}$$

$$+ \frac{\partial G}{\partial \underline{\theta}}\,QG^T\underline{y} - PH^TR^{-1}H\,\frac{\partial P}{\partial \underline{\theta}}\,\underline{y} - PH^TR^{-1}\,\frac{\partial H}{\partial \underline{\theta}}\,P\underline{y} - PH^T\,\frac{\partial R^{-1}}{\partial \underline{\theta}}\,HP\underline{y}$$

$$- P\,\frac{\partial H^T}{\partial \underline{\theta}}\,R^{-1}HP\underline{y} - \frac{\partial P}{\partial \underline{\theta}}\,H^TR^{-1}HP\underline{y} \qquad \text{(B.3)}$$

Apply the transformations:

$$\underline{Y} = \frac{\partial P}{\partial \underline{\theta}}\,\underline{y} \qquad \text{(B.4)}(\underline{3})$$

$$\dot{\underline{Y}} = \frac{\partial P}{\partial \underline{\theta}} \dot{\underline{y}} + \left(\frac{\partial \dot{P}}{\partial \theta}\right) \underline{y} \tag{B.5}$$

By reviewing the order of differentiation for the term $\frac{\partial \dot{P}}{\partial \underline{\theta}} \underline{y}$ in equation(B.3) the equation (B.5) can be substituted in equation (B.3).

$$\begin{aligned} \dot{\underline{Y}} - \frac{\partial P}{\partial \underline{\theta}} \dot{\underline{y}} &= F\underline{Y} + \frac{\partial F}{\partial \underline{\theta}} P\underline{y} \\ &+ P \frac{\partial F^T}{\partial \underline{\theta}} \underline{y} + \frac{\partial P}{\partial \underline{\theta}} F^T \underline{y} + GQ \frac{\partial G^T}{\partial \underline{\theta}} \underline{y} + G \frac{\partial Q}{\partial \underline{\theta}} G^T \underline{y} + \frac{\partial G}{\partial \underline{\theta}} QG^T \underline{y} - PH^T R^{-1} H\underline{Y} \\ &- PH^T R^{-1} \frac{\partial H}{\partial \underline{\theta}} P\underline{y} - PH^T \frac{\partial R^{-1}}{\partial \underline{\theta}} HP\underline{y} - P \frac{\partial H^T}{\partial \underline{\theta}} R^{-1} HP\underline{y} - \frac{\partial P}{\partial \underline{\theta}} H^T R^{-1} HP\underline{y} \end{aligned} \tag{B.6}$$

Apply the transformation

$$\dot{\underline{y}} = -F^T \underline{y} + H^T R^{-1} HP\underline{y} \qquad \text{(B.7)(\underline{3})}$$

$$\begin{aligned} \dot{\underline{Y}} &= F\underline{Y} + \frac{\partial F}{\partial \underline{\theta}} P\underline{y} + P \frac{\partial F^T}{\partial \underline{\theta}} \underline{y} + GQ \frac{\partial G^T}{\partial \underline{\theta}} \underline{y} + G \frac{\partial Q}{\partial \underline{\theta}} G^T \underline{y} + \frac{\partial G}{\partial \underline{\theta}} QG^T \underline{y} \\ &- PH^T R^{-1} H\underline{Y} - PH^T R^{-1} \frac{\partial H}{\partial \underline{\theta}} P\underline{y} - PH^T \frac{\partial R^{-1}}{\partial \underline{\theta}} HP\underline{y} - P \frac{\partial H^T}{\partial \underline{\theta}} R^{-1} HP\underline{y} \end{aligned} \tag{B.8}$$

Equation (B.7) and (B.8) are a linear system and are written in matrix form in B.9.

$$\begin{bmatrix} \dot{\underline{y}} \\ \\ \dot{\underline{Y}} \end{bmatrix} = \begin{bmatrix} \omega_{11} & \omega_{12} \\ \\ \omega_{21} & \omega_{22} \end{bmatrix} \begin{bmatrix} \underline{y} \\ \\ \underline{Y} \end{bmatrix} \tag{B.9}$$

where

$$\omega_{11} = -F^T + H^T R^{-1} HP \tag{B.10}$$

$$\omega_{12} = 0 \tag{B.11}$$

$$\begin{aligned} \omega_{21} &= \frac{\partial F}{\partial \underline{\theta}} P + P \frac{\partial F^T}{\partial \underline{\theta}} + GQ \frac{\partial G^T}{\partial \underline{\theta}} + G \frac{\partial Q}{\partial \underline{\theta}} G^T + \frac{\partial G}{\partial \underline{\theta}} QG^T \\ &- PH^T R^{-1} \frac{\partial H}{\partial \underline{\theta}} P - PH^T \frac{\partial R^{-1}}{\partial \underline{\theta}} HP - P \frac{\partial H^T}{\partial \underline{\theta}} R^{-1} HP \end{aligned} \tag{B.12}$$

$$\omega_{22} = F - PH^T R^{-1} H \tag{B.13}$$

Equation (B.9) can be solved using a state transition matrix approach.

$$\begin{bmatrix} \underline{y}(t_o+\Delta t) \\ \\ \underline{\gamma}(t_o+\Delta t) \end{bmatrix} = \begin{bmatrix} \phi_{yy}(t_o,\Delta t) & \phi_{y\gamma}(t_o,\Delta t) \\ \\ \phi_{\gamma y}(t_o,\Delta t) & \phi_{\gamma\gamma}(t_o,\Delta t) \end{bmatrix} \begin{bmatrix} \underline{y}(t_o) \\ \\ \underline{\gamma}(t_o) \end{bmatrix} \qquad \text{(B.14)}$$

where

$$\Phi(t_o,\Delta t) = \exp \begin{bmatrix} a_{11} & a_{12} \\ \\ a_{21} & a_{22} \end{bmatrix} \Delta t \qquad \text{(B.15)}$$

From equation (B.14)

$$\underline{y}(t_o+\Delta t) = \phi_{yy}(t_o,\Delta t)\ \underline{y}(t_o) + \phi_{y\gamma}(t_o,\Delta t)\ \underline{\gamma}(t_o) \qquad \text{(B.16)}$$

$$\underline{\gamma}(t_o+\Delta t) = \phi_{\gamma y}(t_o,\Delta t)\ \underline{y}(t_o) + \phi_{\gamma\gamma}(t_o,\Delta t)\ \underline{\gamma}(t_o) \qquad \text{(B.17)}$$

Applying equation (B.4) and combining (1.16) and (1.17):

$$\frac{\partial P(t_o+\Delta t)}{\partial \underline{\theta}} = [\phi_{\gamma y}(t_o,\Delta t) + \phi_{\gamma\gamma}(t_o,\Delta t)\ \frac{\partial P(t_o)}{\partial \underline{\theta}}]^{-1}$$

$$[\phi_{yy}(t_o,\Delta t) + \phi_{y\gamma}(t_o,\Delta t)\ \frac{\partial P(t_o)}{\partial \underline{\theta}}]^{-1} \qquad \text{(B.18)}$$

Appendix C

DEFINITIONS OF VARIABLES USED IN $\underline{F}$ AND $\underline{G}$ MATRICES

Parameter	Symbol	Value	Units
Wheelset mass	M	12.045	kg
Nominal longitudinal creep coefficient	f_{11}	8705.5	n
Nominal lateral creep coefficient	f_{22}	2549.0	n
Lateral spring constant	K_y	1490.0	n/m
Yaw spring constant	K_ψ	77.3	n-m/rad
One-half distance between contact points	ℓ	0.147	m
Wheel conicity	α	0.05	rad
Wheelset rolling radius	r_o	0.083	m
Lateral damping constant	c	96.5	n-m-s/rad
Forward velocity	V	3.021	m/s

Appendix D

SIMPLIFICATION OF $\frac{\partial P(t_i)}{\partial \underline{\theta}}$ FOR WHEELSET PROBLEM

Simplifying equations (B.11) through (B.14) from Appendix B, equation (B.10) reduces to:

$$\begin{bmatrix} \dot{\underline{y}} \\ \\ \dot{\underline{Y}} \end{bmatrix} = \begin{bmatrix} -F^T + H^T R^{-1} HP & 0 \\ \\ \frac{\partial F}{\partial \underline{\theta}} P + P \frac{\partial F^T}{\partial \theta} & F - PH^T R^{-1} H \end{bmatrix} \begin{bmatrix} \underline{y} \\ \\ \underline{Y} \end{bmatrix} \qquad (D.1)$$

Equation (D.1) is solved using the state transition matrix approach.

$$\begin{bmatrix} \underline{y}(t_o + \Delta t) \\ \\ \underline{Y}(t_o + \Delta t) \end{bmatrix} = \begin{bmatrix} \phi_{yy}(\Delta t) & \phi_{yY}(\Delta t) \\ \\ \phi_{Yy}(\Delta t) & \phi_{YY}(\Delta t) \end{bmatrix} \begin{bmatrix} \underline{y}(t_o) \\ \\ \underline{Y}(t_o) \end{bmatrix} \qquad (D.2)$$

$$\Phi(\Delta t) = \exp \begin{bmatrix} -F^T + H^T R^{-1} HP & 0 \\ \\ \frac{\partial F}{\partial \underline{\theta}} P + P \frac{\partial F^T}{\partial \underline{\theta}} & F - PH^T R^{-1} H \end{bmatrix} \Delta t \qquad (D.3)$$

Using the same methodology as in Appendix B

$$\frac{\partial P(t_o + \Delta t)}{\partial \underline{\theta}} = [\, \phi_{Yy}(\Delta t) + \phi_{YY}(\Delta t) \frac{\partial P(t_o)}{\partial \underline{\theta}} \,]$$

$$[\, \phi_{yy}(\Delta t) + \phi_{yY}(\Delta t) \frac{\partial P(t_o)}{\partial \underline{\theta}} \,]^{-1} \qquad (D.4)$$

Equation (D.4) is an iterative equation and is solved for $\left.\frac{\partial P}{\partial \underline{\theta}}\right|_{ss}$. Because the wheelset is a time invariant system $\left.\frac{\partial P}{\partial \underline{\theta}}\right|_{ss}$ is solved for only once for each iteration of the maximum likelihood equations.

AD-A107 447 AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH F/G 12/1
IDENTIFICATION OF WHEELSET/RAIL CREEP COEFFICIENTS FROM DYNAMIC--ETC(U)
JAN 79 W N HERZOG
UNCLASSIFIED AFIT-CI-79-254T NL

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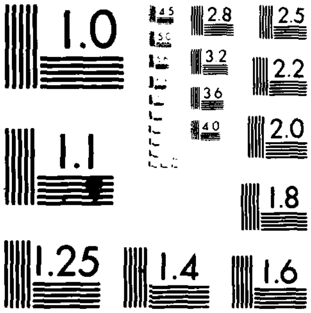

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS-1963-A

Appendix E

LISTING OF THE MAXIMUM LIKELIHOOD PROGRAM

```
      CALL INDUMP
C        THIS PROGRAM IS THE CONTROLLING PROGRAM FOR A SERIES OF
C        SUBROUTINES WHICH FORM THE MAXIMUM LIKELIHOOD PARAMETER
C        IDENTIFICATION PROCESSOR FOR THE SIMULATED WHEELSET DATA.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION DELTAP(3,3)
      DIMENSION DELTAK(3,3)
      DIMENSION DPHIX(3,3)
      DIMENSION DFMTX(3,3)
      DIMENSION ZMEAS(3,1001)
      DIMENSION ZMEAS1(3,1001)
      DIMENSION XHAT(3,1001)
      DIMENSION RNU(3,1001)
      DIMENSION ZHAT(3,1001)
      DIMENSION DXHAT(3,1001)
      DIMENSION XMEAS1(3,1001)
      DIMENSION XMEAS(3,1001)
      DIMENSION XERROR(3,1001)
      DIMENSION XVECT1(1001)
      DIMENSION YVECT1(1001)
      DIMENSION YVECT2(1001)
      DIMENSION U(1,1001)
      READ(5,10) K1F,K1G,K2G,K1H,K2H,K1Q,K1R
   10 FORMAT(I3)
      WRITE(6,15) K1F,K1G,K2G,K1H,K2H,K1Q,K1R
   15 FORMAT(//,' ',5X,'K1F=',I3,5X,'K1G=',I3,5X,'K2G=',I3,5X,'K1H=',
     $I3,5X,'K2H=',I3,5X,'K1Q=',I3,5X,'K1R=',I3)
      K1DELK=K1F
      K2DELK=K1F
      K1DELP=K1F
      K2DELP=K1F
      K1DPH=K1F
      K2DPH=K1F
      K1DF=K1F
      K2DF=K1F
      READ(5,25)IDATSA,IDATEN
   25 FORMAT(I4)
      NPOINT=(IDATEN-IDATSA)+1
      NPT1=NPOINT+1
      IFLAG=0
      WRITE(6,45)IDATSA,IDATEN,NPOINT,NPT1
   45 FORMAT(//,' ',5X,'IDATSA=',I4,5X,'IDATEN=',I4,5X,'NPOINT=',
     $I4,5X,'NPT1=',I4)
      CALL DRIVER(ZMEAS,ZMEAS1,XHAT,RNU,ZHAT,DELTAP,DELTAK,DXHAT,
     $DPHIX,DFMTX,XERROR,XMEAS1,XMEAS,XVECT1,YVECT1,YVECT2,U)
      STOP
      END
C
C
      SUBROUTINE DRIVER(ZMEAS,ZMEAS1,XHAT,RNU,ZHAT,DELTAP,DELTAK,DXHAT,
     $DPHIX,DFMTX,XERROR,XMEAS1,XMEAS,XVECT1,YVECT1,YVECT2,U)
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IROSTA,IROEND,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DOUBLE PRECISICN PHIZ(6,6)
      DIMENSION PHIX(3,3)
      DOUBLE PRECISION PHIXDP(3,3)
      DIMENSION ZMEAS(3,NPOINT)
      DIMENSION ZMEAS1(3,NPT1)
      DIMENSION DXHAT(3,NPOINT)
      DIMENSION RNU(3,NPOINT)
      DIMENSION ZHAT(3,NPOINT)
      DIMENSION DELTAP(K1DELP,K2DELP)
      DIMENSION DELTAK(K1DELK,K2DELK)
      DIMENSION DPHIX(K1DPH,K2DPH)
      DIMENSION DFMTX(K1DF,K2DF)
      DIMENSION Z(6,6)
      DIMENSION XHAT(3,NPOINT)
      DIMENSION XMEAS1(3,NPT1)
      DIMENSION XMEAS(3,NPOINT)
      DIMENSION XERRCR(3,NPOINT)
      DIMENSION XVECT1(NPOINT)
      DIMENSION YVECT1(NPOINT)
      DIMENSION YVECT2(NPOINT)
      DOUBLE PRECISICN PHIK(3,3)
      DIMENSION ITIT1(10)
      DIMENSION ITIT2(10)
      DIMENSION JLAB1(10)
      DIMENSION JLAB2(10)
      DIMENSION JLAB3(10)
      DIMENSION ITIT3(10)
      DIMENSION VDELJ(20)
      DIMENSION VDELJ2(20)
      DIMENSION VRLIKE(20)
      DIMENSION VTHETA(2C)
      DIMENSION ITIT5(10)
      DIMENSION ITIT4(10)
      DIMENSION JLAB4(10)
      DIMENSION JLAB5(10)
      DIMENSION ITIT6(10)
      DIMENSION JLAB6(10)
      DIMENSION JLAB7(10)
      DIMENSION ITIT7(10)
      DIMENSION VCONST(20)
      DIMENSION VDETB(20)
      DIMENSION U(1,NPT1)
      TVEL=3.021
      CALL PARAM(THETA)
      WRITE(6,20) RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,THETA,BZERO,
     $DT,SDT
 20   FORMAT(//,' ',2X,'RMASS=',F12.5,2X,'F11=',F12.5,2X,'F22=',F12.5,
     $2X,'RKY=',F12.5,2X,'RKSI=',F12.5,2X,'RLE=',F12.5,//,
     $' ',2X,'ALPHA=',F12.5,2X,'RZERO=',F12.5,2X,'THETA=',
     $F12.5,2X,'BZERC=',F12.5,2X,'DT=',F12.5,2X,'SDT=',F12.5)
      WRITE(6,40)
 4C   FORMAT(/,' ',12X,'G MATRIX')
      CALL OUTPUT(G,K1G,K2G)
      WRITE(6,60)
 60   FORMAT(/,' ',12X,'H MATRIX')
      CALL OUTPUT(H,K1H,K2H)
      WRITE(6,80)
 80   FORMAT(/,' ',12X,'Q MATRIX')
      CALL OUTPUT(Q,K1Q,K1Q)
      WRITE(6,100)
 100  FORMAT(/,' ',12X,'R MATRIX')
      CALL OUTPUT(R,K1R,K1R)
      DO 120 I=1,NPT1
      READ(5,140) (ZMEAS1(J,I),J=1,3)
 120   CONTINUE
 140  FORMAT(3F15.8)
      DO 160 J=1,3
      I=IDATSA-1
      XIC(J,1)=ZMEAS1(J,I)
 160  CONTINUE
      WRITE(6,180)
 180  FORMAT(/,' ',12X,'XIC')
      CALL OUTPUT(XIC,3,1)
      DO 200 J=1,3
      DO 200 I=IDATSA,IDATEN
      M=I-1
      ZMEAS(J,M)=ZMEAS1(J,I)
 200  CONTINUE
      READ(5,210)ITIT1
 210  FORMAT(10A4)
      READ(5,210)JLAB1
      READ(5,210)ITIT2
      READ(5,210)JLAB2
      READ(5,210)ITIT3
      READ(5,210)JLAB3
      READ(5,210)ITIT4
      READ(5,210)JLAB4
      READ(5,210)ITIT5
      READ(5,210)JLAB5
      READ(5,210)ITIT6
      READ(5,210)JLAB6
      READ(5,210)ITIT7
      READ(5,210)JLAB7
      DO 212 J=1,NPT1
 212  READ(5,213)U(1,J)
 213  FORMAT(E15.8)
      DO 800 ISTEP=1,20
      WRITE(6,215)ISTEP
 215  FORMAT(////,' ',1X,'ISTEP=',I3)
      WRITE(6,216)THETA
 216  FORMAT(/,' ',2X,'THETA=',F15.8)
      CALL SYSMTX(THETA)
      WRITE(6,220)
 220  FORMAT(/,' ',12X,'F MATRIX')
      CALL OUTPUT(F,K1F,K1F)
      CALL ZMTRX(Z,K1Z)
      WRITE(6,240)
 240  FORMAT(/,' ',12X,'Z MATRIX')
      CALL OUTPUT(Z,K1Z,K1Z)
      CALL STMZ(Z,DT,PHIZ,K1Z,K1PHIZ,K2PHIZ)
      IF(IFLAG .EQ. 1)GO TO 1000
      WRITE(6,260)
 260  FORMAT(/,' ',12X,'KALMAN FILTER STM (PHIZ)')
      CALL OUTDP(PHIZ,6,6)
      CALL RICATI(PHIZ,K1PHIZ,P,K1P)
      IF(IFLAG .EQ. 1)GO TO 1000
      WRITE(6,280)
 280  FORMAT(/,' ',12X,'STATE COVARIANCE MATRIX (P)')
      CALL OUTPUT(P,K1P,K1P)
      CALL KGMTRX
      WRITE(6,300)
 300  FORMAT(/,' ',12X,'KALMAN GAIN MATRIX (RKGM)')
      CALL OUTPUT(RKGM,K1RKGM,K2RKGM)
      CALL STMP(P,DT,PHIXDP,K1P,K1PHIX,K2PHIX)
      CALL DPTOSP(PHIXDP,K1PHIX,K2PHIX,PHIX)
      WRITE(6,320)
 320  FORMAT(/,' ',12X,'SYSTEM STATE TRANSITION MATRIX (PHIX)')
      CALL OUTPUT(PHIX,K1PHIX,K2PHIX)
      CALL STATE(ZMEAS,XHAT,PHIK,K1PHIK,U)
 339  CALL RINOVA(XHAT,ZMEAS,ZHAT,RNU)
      CALL ZCOV
      WRITE(6,340)
 340  FORMAT(/,' ',12X,'MEASUREMENT COVARIANCE MATRIX (B)')
      CALL OUTPUT(B,3,3)
      CALL RMLPI(RNU,SC,SD)
      VCONST(ISTEP)=SC
      VDETB(ISTEP)=SD
      WRITE(6,360)
 360  FORMAT(/,' ',12X,'VALUE OF THE LIKELIHOOD FUNCTION (RLIKE)')
      WRITE(6,380) RLIKE
 380  FORMAT(' ',12X,'RLIKE=',E15.8)
      CALL PARP(THETA,DPMTX)
      WRITE(6,400)
 400  FORMAT(/,' ',12X,'PARTIAL OF P WRT THETA (DPMTX)')
      CALL OUTPUT(DPMTX,K1DP,K2DP)
      CALL PDP(DPMTX,DELTAP,DELTAK)
      WRITE(6,440)
 440  FORMAT(/,' ',12X,'PARTIAL OF P WRT THETA (DELTAP)')
      CALL OUTPUT(DELTAP,K1DELP,K2DELP)
      WRITE(6,460)
 460  FORMAT(/,' ',12X,'PARTIAL OF RKGM WRT THETA (DELTAK)')
      CALL OUTPUT(DELTAK,K1DELK,K2DELK)
      CALL XSEN(PHIK,K1PHIK,DPMTX,DELTAK,XHAT,ZMEAS,DXHAT)
      CALL GRADE(DXHAT,RNU,DELTAP,DELTAJ)
      WRITE(6,480)
 480  FORMAT(/,' ',12X,'GRADIENT - PARTIAL OF THE LIKELIHOOD FUNCTION',
     $'WRT THETA')
      WRITE(6,500) DELTAJ
 500  FORMAT(' ',12X,'DELTAJ=',E15.8)
      CALL FISHER(RNU,DXHAT,DELTAP,DELJ2)
      WRITE(6,520)
 520  FORMAT(/,' ',12X,'FISHER INFORMATION MATRIX')
      WRITE(6,540)DELJ2
 540  FORMAT(' ',12X,'DELJ2=',E15.8)
      CALL STEP(THETA,DELTAJ,DELJ2,DTHETA,THETAN)
      WRITE(6,550) DTHETA
 550  FORMAT(//,' ',12X,'DTHETA=',E15.8)
      WRITE(6,560) THETAN
 560  FORMAT(/,' ',12X,'THETAN=',E15.8)
      VTHETA(ISTEP)=THETA
      VDELJ(ISTEP)=DELTAJ
      VDELJ2(ISTEP)=DELJ2
      VRLIKE(ISTEP)=RLIKE
      THETA=THETA+0.05
 800  CONTINUE
 805  CONTINUE
```

```
      WRITE(6,810)ITIT3
 810  FORMAT('1',40X,10A4)
      CALL WPLOT1(VTHETA,VRLIKE,ISTEP,40,JLAB3)
      WRITE(6,810)ITIT1
      CALL WPLOT1(VTHETA,VCONST,ISTEP,40,JLAB1)
      WRITE(6,810)ITIT6
      CALL WPLOT1(VTHETA,VDETB,ISTEP,40,JLAB6)
      WRITE(6,810)ITIT4
      CALL WPLOT1(VTHETA,VDELJ,ISTEP,40,JLAB4)
      WRITE(6,810)ITIT5
      CALL WPLOT1(VTHETA,VDELJ2,ISTEP,40,JLAB5)
 900  CONTINUE
 1000 RETURN
      END
C
C
      SUBROUTINE PARAM(THETA)
C        SUBROUTINE PARAM READS IN THE PARAMETERS NECESSARY TO FORM
C        THE SYSTEM MATRIX
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
 10   FORMAT(F15.8)
      READ(5,10) RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,THETA,BZERO,DT,
     $SDT
      DO 20 I=1,K1G
      DO 20 J=1,K2G
 20   READ(5,10) G(I,J)
      DO 30 I=1,K1H
      DO 30 J=1,K2H
 30   READ(5,10) H(I,J)
      DO 40 I=1,K1Q
      DO 40 J=1,K1Q
 40   READ(5,10) Q(I,J)
      DO 50 I=1,K1R
      DO 50 J=1,K1R
 50   READ(5,10) R(I,J)
      DO 70 I=1,4
      DO 70 J=1,3
      READ(5,10) ATTEN(I,J)
 70   CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SYSMTX(THETA)
C        SUBROUTINE SYSMTX FORMS THE SYSTEM MATRIX - F MATRIX
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      F(1,1)=-((((2.0)*F22*THETA)+(BZERO*TVEL))/(RMASS*TVEL))
      F(1,2)=2.0*F22*THETA/RMASS
      F(1,3)=(-RKY)/RMASS
      F(2,1)=0.0
      F(2,2)=(-RKSI)*TVEL/(2.0*F11*THETA*(RIE**2.0))
      F(2,3)=(-ALPHA)*TVEL/(RIE*BZERO)
      F(3,1)=1.0
      F(3,2)=0.0
      F(3,3)=0.0
      RETURN
      END
C
C
      SUBROUTINE ZMTRX(Z,K1Z)
C        SUBROUTINE ZMTRX FORMS THE Z MATRIX FOR THE KALMAN ENGLER
C        SOLUTION TO THE RICCATI EQUATION
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
      DIMENSION Z(6,6)
      DIMENSION FT(3,3)
      DIMENSION Z11(3,3)
      DIMENSION RI(3,3)
      DIMENSION HT(3,3)
      DIMENSION RIH(3,3)
      DIMENSION Z12(3,3)
      DIMENSION GT(1,3)
      DIMENSION QGT(3,3)
      DIMENSION  Z21(3,3)
      DIMENSION Z22(3,3)
      DOUBLE PRECISION WKAREA(9)
      DOUBLE PRECISION RIDP(3,3)
      DOUBLE PRECISION RDP(3,3)
 01   CALL TRANS (F,FT,K1F,K1F,K1FT,K2FT)
      CALL NEG(FT,Z11,K1FT,K2FT,K1Z11,K2Z11)
      IDGT=0
      CALL SPTODP(R,K1R,K1R,RDP)
      CALL LINV1F(RDP,K1R,K1R,RIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(RIDP,K1R,K1R,RI)
      CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL MULT(RI,H,RIH,K1R,K1R,K1H,K2H,K1RIH,K2RIH)
      CALL MULT(HT,RIH,Z12,K1HT,K2HT,K1RIH,K2RIH,K1Z12,K2Z12)
      CALL TRANS(G,GT,K1G,K2G,K1GT,K2GT)
      CALL MULT(Q,GT,QGT,K1Q,K1Q,K1GT,K2GT,K1QGT,K2QGT)
      CALL MULT(G,QGT,Z21,K1G,K2G,K1QGT,K2QGT,K1Z21,K2Z21)
      CALL SCALAR(F,1.0,Z22,K1F,K1F,K1Z22,K2Z22)
      DO 10 I=1,K1Z11
      DO 10 J=1,K2Z11
      M=I
      N=J
 10   Z(M,N)=Z11(I,J)
      DO 20 I=1,K1Z12
      DO 20 J=1,K2Z12
      M=I
      N=J+K2Z11
 20   Z(M,N)=Z12(I,J)
      DO 30 I=1,K1Z21
      DO 30 J=1,K2Z21
      M=I+K1Z11
      N=J
   30 Z(M,N)=Z21(I,J)
      DO 40 I=1,K1Z22
      DO 40 J=1,K2Z22
      M=I+K1Z21
      N=J+K2Z12
   40 Z(M,N)=Z22(I,J)
      K1Z=K1Z11+K1Z21
      RETURN
      END
C
C
      SUBROUTINE STMF(F,DT,PHI,K1F,K1PHI,K2PHI)
C        SUBROUTINE STMF DIMENSIONS THE NECESSARY ARRAYS FOR
C        SUBROUTINE STM.  IT ACTS AS A DRIVER FOR SUBROUTINE STM.
      DOUBLE PRECISION FDP(3,3)
      DIMENSION F(K1F,K1F)
      DOUBLE PRECISION PHI(K1F,K1F)
      DOUBLE PRECISION FDT(3,3)
      DOUBLE PRECISION PRD(3,3)
      DOUBLE PRECISION PRD1(3,3)
      DOUBLE PRECISION TERM(3,3)
      DOUBLE PRECISION PHI1(3,3)
      DOUBLE PRECISION DIFF(3,3)
      DOUBLE PRECISION  RM(3,3)
   01 CALL SPTODP(F,K1F,K1F,FDP)
      CALL STM(FDP,PHI,FDT,PRD,PRD1,TERM,PHI1,DIFF,RM,DT,K1F,
     $K1PHI,K2PHI,IFLAG)
      RETURN
      END
C
C
      SUBROUTINE STMZ(Z,DT,PHI,K1Z,K1PHIZ,K2PHIZ)
C        SUBROUTINE STMZ DIMENSIONS THE NECESSARY ARRAYS FOR SUB-
C        ROUTINE STM.  IT ACTS AS A DRIVER FOR SUBROUTINE STM
      DIMENSION Z(K1Z,K1Z)
      DOUBLE PRECISION PHI(6,6)
      DOUBLE PRECISION FDT(6,6)
      DOUBLE PRECISION PRD(6,6)
      DOUBLE PRECISION PRD1(6,6)
      DOUBLE PRECISION TERM(6,6)
      DOUBLE PRECISION ZDP(6,6)
      DOUBLE PRECISION  PHI1(6,6)
      DOUBLE PRECISION  DIFF(6,6)
      DOUBLE PRECISION RM(6,6)
   01 CALL SPTODP(Z,K1Z,K1Z,ZDP)
      CALL STM(ZDP,PHI,FDT,PRD,PRD1,TERM,PHI1,DIFF,RM,DT,K1Z,
     $K1PHIZ,K2PHIZ,IFLAG)
      RETURN
      END
C
C
      SUBROUTINE STM(F,PHI,FDT,PRD,PRD1,TERM,PHI1,DIFF,RM,DT,K1F,
     $K1PHI,K2PHI,IFLAG)
C        SUBROUTINE STM CALCULATES THE STATE TRANSITION MATRIX.
      DOUBLE PRECISION F(K1F,K1F)
      DOUBLE PRECISION FDT(K1F,K1F)
      DOUBLE PRECISION PHI(K1F,K1F)
      DOUBLE PRECISION PRD(K1F,K1F)
      DOUBLE PRECISION PHI1(K1F,K1F)
      DOUBLE PRECISION PRD1(K1F,K1F)
      DOUBLE PRECISION DIFF(K1F,K1F)
      DOUBLE PRECISION TERM(K1F,K1F)
      DOUBLE PRECISION RM(K1F,K1F)
      DOUBLE PRECISION B
      DOUBLE PRECISION COEF
      DOUBLE PRECISION CT
      DOUBLE PRECISION CHK
      DOUBLE PRECISION DTDP
      DOUBLE PRECISION FACTDP
 01   DTDP=DBLE(DT)
      CT=DBLE(1.0E-16)
      IFLAGO=0
      B=DBLE(0.0)
      CALL SCALDP(F,B,RM,K1F,K1F,K1RM,K2RM)
      DO 20 I=1,K1RM
 20   RM(I,I)=DBLE(1.C)
      DO 320 I=1,200
      IF(I .GT. 1)GO TO 120
      CALL SCALDP(F,DTDP,FDT,K1F,K1F,K1FD1,K2FDT)
      CALL ADDDP(RM,FDT,PHI,K1RM,K2RM,K1PHI,K2PHI)
      B=DBLE(1.0)
      CALL SCALDP(F,B,PRD,K1F,K1F,K1PRD,K2PRD)
      GO TO 300
 120  CALL MULTDP(F,PRD,PRD1,K1F,K1F,K1PRD,K2PRD,K1PRD1,K2PRD1)
      CHK=DBLE(1.0E 71)
      CALL OVERDP(PRD1,CHK,K1PRD1,K2PRD1,IFLAGO)
      IF(IFLAGO .EQ. 0)GO TO 160
      WRITE(6,125)
 125  FORMAT(/,' ',2X,'PRD1 EXCEEDS EXPONENT OVERFLOW.',
     $1X,'CURRENT VALUE OF PHI USED.  EXECUTION CONTINUES.')
      GO TO 420
 160  B=DBLE(1.0)
      CALL SCALDP(PRD1,B,PRD,K1PRD1,K2PRD1,K1PRD,K2PRD)
      COEF=(DT**I)/(FACTDP(I))
      CHK=DBLE(1.0E-76)
      IF(COEF .LE. CHK)GO TO 380
      CALL SCALDP(PRD,COEF,TERM,K1PRD,K2PRD,K1TERM,K2TERM)
      CALL ADDDP(PHI,TERM,PHI,K1PHI,K2PHI,K1PHI,K2PHI)
      CALL SUBDP(PHI,PHI1,DIFF,K1PHI,K2PHI,K1DIFF,K2DIFF)
      DO 260 J=1,K1PHI
      DO 260 K=1,K2PHI
      IF(DABS(DIFF(J,K)) .GT. CT)GO TO 280
 260  CONTINUE
      GO TO 420
 280  IF(I-200) 300,340,340
 300  B=DBLE(1.0)
      CALL SCALDP(PHI,B,PHI1,K1PHI,K2PHI,K1PHI1,K2PHI1)
 320  CONTINUE
 340  WRITE(6,360)
 360  FORMAT(//,' ',5X,'200 ITERATIONS. NO CONVERGENCE')
      GO TO 450
 380  WRITE(6,400)
 400  FORMAT(//,' ',2X,'COEF .LE. 1.0E-76. ITERATION STOPPED.',
     $1X,'CURRENT VALUE OF PHI USED. EXECUTION CONTINUES.')
 420  WRITE(6,440)I
 440  FORMAT(//,' ',2X,I3,1X,'ITERATIONS FOR STM.')
 450  WRITE(6,460)
 460  FORMAT(/,' ',12X,'STM DIFF MATRIX')
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```
      CALL OUTDP(DIFF,K1DIFF,K2DIFF)
  480 RETURN
      END
C
C
      SUBROUTINE RICATI(PHIZ,K1PHI,PSP,K1P)
C        SUBROUTINE RICCATI CALCULATES THE STATE COVARIANCE MATRIX I.E.
C        THE P MATRIX.
      DOUBLE PRECISION PHI11(3,3)
      DOUBLE PRECISION PHI12(3,3)
      DOUBLE PRECISION PHI21(3,3)
      DOUBLE PRECISION PHI22(3,3)
      DOUBLE PRECISION W(3,3)
      DOUBLE PRECISION X(3,3)
      DOUBLE PRECISION Y(3,3)
      DOUBLE PRECISION Z(3,3)
      DOUBLE PRECISION DIFF(3,3)
      DOUBLE PRECISION P1(3,3)
      DOUBLE PRECISION P2(3,3)
      DOUBLE PRECISION P2T(3,3)
      DOUBLE PRECISION ZI(3,3)
      DOUBLE PRECISION PHIZ(K1PHI,K1PHI)
      DOUBLE PRECISION PSUM(3,3)
      DOUBLE PRECISION P(3,3)
      DIMENSION PSP(3,3)
      DOUBLE PRECISION CONST
      DOUBLE PRECISION ERROR
      DOUBLE PRECISION WKAREA(36)
      DOUBLE PRECISION A,B
   01 ERROR=DBLE(1.0E-16)
      K11=K1PHI/2.0
      DO 20 I=1,K11
      DO 20 J=1,K11
   20 PHI11(I,J)=PHIZ(I,J)
      DO 40 I=1,K11
      DO 40 J=1,K11
      M=I
      N=K11+J
   40 PHI12(I,J)=PHIZ(M,N)
      DO 60 I=1,K11
      DO 60 J=1,K11
      M=I+K11
      N=J
   60 PHI21(I,J)=PHIZ(M,N)
      DO 80 I=1,K11
      DO 80 J=1,K11
      M=K11+I
      N=K11+J
   80 PHI22(I,J)=PHIZ(M,N)
      A=DBLE(0.0)
      CALL SCALDP(PHI11,A,P2,K11,K11,K1P2,K2P2)
      ICOUNT=1
   85 B=DBLE(1.0)
      CALL SCALDP(P2,B,P1,K1P2,K2P2,K1P1,K2P1)
      CALL MULTDP(PHI22,P2,W,K11,K11,K1P2,K2P2,K1W,K2W)
      CALL ADDDP(PHI21,W,X,K11,K11,K1X,K2X)
      CALL MULTDP(PHI12,P2,Y,K11,K11,K1P2,K2P2,K1Y,K2Y)
      CALL ADDDP(PHI11,Y,Z,K11,K11,K1Z,K2Z)
      IDGT=0
      CALL LINV1F(Z,K1Z,K11,ZI,IDGT,WKAREA,IER)
      CALL MULTDP(X,ZI,P2,K1X,K2X,K1Z,K2Z,K1P2,K2P2)
      CALL TRANSD(P2,P2T,K1P2,K2P2,K1P2T,K2P2T)
      CALL ADDDP(P2,P2T,PSUM,K1P2,K2P2,K1PSUM,K2PSUM)
      CONST=DBLE(.500)
      CALL SCALDP(PSUM,CONST,P2,K1PSUM,K2PSUM,K1P2,K2P2)
      CALL SUBDP(P2,P1,DIFF,K1P2,K2P2,K1DIFF,K2DIFF)
      DO 100 I=1,K1DIFF
      DO 100 J=1,K1DIFF
   90 IF(DABS(DIFF(I,J))-ERROR) 100,100,110
  100 CONTINUE
      GO TO 150
  110 ICOUNT=ICOUNT+1
      IF(ICOUNT .GT. 200) GO TO 115
      GO TO 85
  115 WRITE(6,120)
  120 FORMAT(//,' ',5X,'200 ITERATIONS. RICCATI SOLUTION DOES NOT',
     $' CONVERGE.')
      B=DBLE(1.0)
  150 CALL SCALDP(P2,B,P,K1P2,K2P2,K1P,K2P)
      CALL DPTOSP(P,K1P,K1P,PSP)
      IF(ICOUNT .GE. 200) GO TO 165
      WRITE(6,160)ICOUNT
  160 FORMAT(/,' ',5X,I3,1X,'ITERATIONS FOR RICCATI SOLUTION TO',
     $' CONVERGE.')
  165 WRITE(6,170)
  170 FORMAT(//,' ',12X,'RICCATI DIFFERENCE MATRIX.')
      CALL OUTDP(DIFF,K1DIFF,K2DIFF)
      RETURN
      END
C
C
      SUBROUTINE KGMTRX
C        SUBROUTINE KGMTRX CALCULATES THE KALMAN GAIN MATRIX.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IROSTA,IROEND,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SIT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPI1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION HTRI(3,3)
      DIMENSION HT(3,3)
      DIMENSION RI(3,3)
      DOUBLE PRECISION WKAREA(9)
      DOUBLE PRECISION RIDP(3,3)
      DOUBLE PRECISION RDP(3,3)
   01 IDGT=0
      CALL SPTODP(R,K1R,K1R,RDP)
      CALL LINV1F(RDP,K1R,K1R,RIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(RIDP,K1R,K1R,RI)
      CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL MULT(HT,RI,HTRI,K1HT,K2HT,K1R,K1R,K1HTRI,K2HTRI)
      CALL MULT(P,HTRI,RKGM,K1P,K1P,K1HTRI,K2HTRI,K1RKGM,K2RKGM)
      RETURN
      END
C
C
      SUBROUTINE STABLE
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION RH(3,3)
      DIMENSION FKMTX(3,3)
      DOUBLE PRECISION WKAREA(6)
      COMPLEX*16 EIGVEC(3,3)
      COMPLEX*16 WDP(3)
      DOUBLE PRECISION FKDP(3,3)
      CALL MULT(RKGM,H,RH,K1RKGM,K2RKGM,K1H,K2H,K1RH,K2RH)
      CALL SUB(F,RH,FKMTX,K1F,K1F,K1FK,K2FK)
      WRITE(6,10)
 10   FORMAT(//,' ',12X,'KALMAN FILTER SYSTEM MATRIX - (F-KH)')
      CALL OUTPUT(FKMTX,K1FK,K2FK)
      IJOB=0
      CALL SPTODP(FKMTX,K1F,K1F,FKDP)
      WRITE(6,15)IER
 15   FORMAT(//,' ',7X,'IER=',I4)
      WRITE(6,20)
 20   FORMAT(//,' ',12X,'EIGENVALUES OF (F-KH)')
      DO 40 I=1,K1FK
 40   WRITE(6,60) WDP(I)
 60   FORMAT(' ',14X,2D15.8)
      DO 80 I=1,K1FK
      IF(REAL(WDP(I)) .LE. 0) GO TO 120
 80   CONTINUE
      WRITE(6,100)
 100  FORMAT(//,' ','***** FILTERED SYSTEM IS UNSTABLE *****')
      IFLAG=1
 120  RETURN
      END
C
C
      SUBROUTINE STATE(ZMEAS,XHAT,PHIK2,K1PHIK,U)
C        SUBROUTINE STATE CALCULATES THE STATE ESTIMATES OF THE
C        KALMAN FILTER ACCORDING TO THE CONTINUOUS FORM OF THE
C        KALMAN FILTER.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION ZMEAS(3,NPOINT)
      DIMENSION  XHAT(3,NPOINT)
      DIMENSION RH(3,3)
      DOUBLE PRECISION WKAREA(36)
      DIMENSION PHIKI(3,3)
      DIMENSION HNEG(3,3)
      DOUBLE PRECISION PHIK(3,3)
      DIMENSION PHIKSP(3,3)
      DIMENSION FKF(3,3)
      DIMENSION FKFI(3,3)
      DIMENSION  RM(3,3)
      DIMENSION T2(3,3)
      DOUBLE PRECISION PHIK2(3,3)
      DOUBLE PRECISION FACTOR
      DIMENSION PRD1(3,3)
      DIMENSION PRD2(3,3)
      DIMENSION XM1(3,1)
      DIMENSION XE1(3,1)
      DIMENSION XE2(3,1)
      DIMENSION GAMMA(3,3)
      DIMENSION X1(3,1)
      DIMENSION X2(3,1)
      DOUBLE PRECISION FKFDP(3,3)
      DOUBLE PRECISION FKFIDP(3,3)
      DOUBLE PRECISION PHIKID(3,3)
      DIMENSION U(1,NPT1)
      DIMENSION U1(1,1)
      DIMENSION A1(3,1)
      DIMENSION A2(3,1)
      DIMENSION LAMBDA(3,1)
      DIMENSION X3(3,1)
      DIMENSION XEP(3,1)
      CALL NEG(H,HNEG,K1H,K2H,K1HNEG,K2HNEG)
      CALL MULT(RKGM,HNEG,RH,K1RKGM,K2RKGM,K1HNEG,K2HNEG,K1RH,K2RH)
      CALL ADD(F,RH,FKF,K1F,K1F,K1FKF,K2FKF)
      WRITE(6,5)
 5    FORMAT(//,' ',12X,'KALMAN FILTER SYSTEM MATRIX (FKF)')
      CALL OUTPUT(FKF,K1FKF,K2FKF)
      CALL STMF(FKF,DT,PHIK,K1FKF,K1PHIK,K2PHIK)
      FACTOR=1.0D 00
      CALL SCALDP(PHIK,FACTOR,PHIK2,K1PHIK,K1PHIK,K1PHK2,K2PHK2)
      CALL DPTOSP(PHIK,K1PHIK,K2PHIK,PHIKSP)
      WRITE(6,10)
 10   FORMAT(//,' ',12X,'STM FOR KALMAN FILTER SYSTEM (PHIK)')
      CALL OUTDP(PHIK,K1PHIK,K2PHIK)
      IDGT=0
      CALL LINV1F(PHIK,K1PHIK,K1PHIK,PHIKID,IDGT,WKAREA,IER)
      CALL DPTOSP(PHIKID,K1PHIK,K2PHIK,PHIKI)
      IDGT=0
      CALL SPTODP(FKF,K1FKF,K2FKF,FKFDP)
      CALL LINV1F(FKFDP,K1FKF,K1FKF,FKFIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(FKFIDP,K1FKF,K2FKF,FKFI)
      CALL SCALAR(F,0.0,RM,K1F,K1F,K1RM,K2RM)
      DO 20 I=1,K1RM
      RM(I,I)=1.0
 20   CONTINUE
      CALL SUB(RM,PHIKI,T2,K1RM,K2RM,K1T2,K2T2)
      CALL MULT(T2,RKGM,PRD1,K1T2,K2T2,K1RKGM,K2RKGM,K1PRD1,K2PRD1)
      CALL MULT(FKFI,PRD1,PRD2,K1FKF,K2FKF,K1PRD1,K2PRD1,K1PRD2,K2PRD2)
      CALL MULT(PHIKSP,PRD2,GAMMA,K1PHIK,K2PHIK,K1PRD2,K2PRD2,K1GAM,
     $K2GAM)
      CALL MULT(T2,G,A1,K1T2,K2T2,K1G,K2G,K1A1,K2A1)
      CALL MULT(FKFI,A1,A2,K1FKF,K2FKF,K1A1,K2A1,K1A2,K2A2)
      CALL MULT(PHIKSP,A2,LAMBDA,K1PHIK,K1PHIK,K1A2,K2A2,K1LAMB,K2LAMB)
      CALL SCALAR(XIC,1.0,XE1,3,1,K1XE1,K2XE1)
      DO 1000 I=1,NPOINT
      DO 100 J=1,3
      XM1(J,1)=ZMEAS(J,I)
 100  CONTINUE
      U1(1,1)=U(1,I)
      CALL MULT(PHIKSP,XE1,X1,K1PHIK,K2PHIK,K1XE1,K2XE1,K1X1,K2X1)
      CALL MULT(GAMMA,XM1,X2,K1GAM,K2GAM,3,1,K1X2,K2X2)
      CALL MULT(LAMBDA,U1,X3,K1LAMB,K2LAMB,1,1,K1X3,K2X3)
      CALL ADD(X1,X2,XEP,K1X1,K2X1,K1XEP,K2XEP)
      CALL ADD(XEP,X3,XE2,K1XEP,K2XEP,K1XE2,K2XE2)
      DO 200 J=1,3
      XHAT(J,I)=XE2(J,1)
  200 CONTINUE
      CALL SCALAR(XE2,1.0,XE1,K1XE2,K2XE2,K1XE1,K2XE1)
 1000 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE ZCOV
C         SUBROUTINE ZCOV CALCUALTES THE B MATRIX FOR THE MAXIMUM
C         LIKELIHOOD PARAMETER IDENTIFICATION PROCESSOR.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IROSTA,IROEND,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
      DIMENSION HT(3,3),PHT(3,3),HPHT(3,3)
   01 CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL MULT(P,HT,PHT,K1P,K1P,K1HT,K2HT,K1PHT,K2PHT)
      CALL MULT(H,PHT,HPHT,K1H,K2H,K1PHT,K2PHT,K1HPHT,K2HPHT)
      CALL ADD(HPHT,R,B,K1HPHT,K2HPHT,K1B,K2B)
      RETURN
      END
C
C
      SUBROUTINE RINCVA(XHAT,ZMEAS,ZHAT,RNU)
C         SUBROUTINE RINCVA CALCULATES THE INNOVATION FOR THE MLPI
C         PROCESSOR.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IROSTA,IROEND,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
      DIMENSION XHAT(3,NPOINT)
      DIMENSION ZMEAS(3,NPOINT)
      DIMENSION RNU(3,NPOINT)
      DIMENSION ZHAT(3,NPOINT)
   01 CALL MULT(H,XHAT,ZHAT,K1H,K2H,3,NPOINT,K1ZHAT,K2ZHAT)
      CALL SUB(ZMEAS,ZHAT,RNU,3,NPOINT,K1RNU,K2RNU)
      RETURN
      END
C
C
      SUBROUTINE RMLPI(RNU,SC,SD)
C         SUBROUTINE RMLPI CALCULATES THE VALUE OF THE LIKELIHOOD FUNC-
C         TION.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IROSTA,IROEND,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
      DIMENSION RNU(3,NPOINT)
      DIMENSION C(3,1)
      DIMENSION ATC(1,1)
      DIMENSION AT(1,3)
      DIMENSION A(3,1)
      DIMENSION BI(3,3)
      DOUBLE PRECISION EX
      DOUBLE PRECISION WKAREA(9)
      DOUBLE PRECISION BIDP(3,3)
      DOUBLE PRECISION DETBDP
      DOUBLE PRECISION BDP(3,3)
      DOUBLE PRECISION VECTOR(3)
      DOUBLE PRECISION D1DP
      DOUBLE PRECISION D2DP
      DOUBLE PRECISION BDP1(3,3)
      DOUBLE PRECISION FACTOR
 01   SUM=0.0
      SC=0.0
      SD=0.0
      IDGT=0
      CALL SPTODP(B,K1B,K1B,BDP)
      FACTOR=1.00D 00
      CALL SCALDP(BDP,FACTOR,BDP1,K1B,K1B,K1BDP1,K2BDP1)
      CALL LINV1P(BDP,K1B,K1B,BIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(BIDP,K1B,K1B,BI)
      IJOB=4
      D1DP=0.00D 00
      CALL LINV3P(BDP1,VECTOR,IJOB,K1B,K1B,D1DP,D2DP,WKAREA,IER)
      IF(D2DP .GE. 0.00D 00) GO TO 50
      D2DP=-D2DP
      EX=2.00D 00**D2DP
      DETBDP=D1DP/EX
      GO TO 60
 50   EX=2.00D 00**D2DP
      DETBDP=D1DP*EX
 60   DETB=DETBDP
      S1=ALOG(DETB)
      WRITE(6,65) S1
 65   FORMAT(//,' ',12X,'NATURAL LOG OF THE DETERMINANT OF B=',
     $1X,E15.8)
      DO 100 I=1,NPCINT
      DO 80 J=1,3
      A(J,1)=RNU(J,I)
 80   CONTINUE
      CALL MULT(BI,A,C,3,3,3,1,K1C,K2C)
      CALL TRANS(A,AT,3,1,K1AT,K2AT)
      CALL MULT(AT,C,ATC,K1AT,K2AT,K1C,K2C,K1ATC,K2ATC)
      CONST=ATC(1,1)
      SC=SC+CONST
      SD=SD+ALOG(DETB)
      SUM=SUM+(CONST+ALOG(DETB))
 100  CONTINUE
      SD=(-1.0/2.0)*SD
      SC=(-1.0/2.0)*SC
      RLIKE=(-1.0/2.0)*SUM
      RETURN
      END
C
C
      SUBROUTINE PARPHI(DPMTX,DPHIX)
C        SUBROUTINE PARPHI CALCULATES THE PARTIAL DERIVATIVE OF THE
```

```
C        STATE TRANSITION MATRIX (PHI) WITH RESPECT TO THETA.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DATA ERROR/.000001/
      DIMENSION PRD(3,3)
      DIMENSION PRD1(3,3)
      DIMENSION TERM(3,3)
      DIMENSION SUM(3,3)
      DIMENSION DFMTX(K1DF,K2DF)
      DIMENSION DPHIX(K1DPH,K2DPH)
      DIMENSION PRD2(3,3)
      DIMENSION DIFF(3,3)
 01   CALL SCALAR(F,0.0,DPHIX,K1F,K1F,K1DPH,K2DPH)
      CALL SCALAR(F,1.0,PRD,K1F,K1F,K1PRD,K2PRD)
      DO 100 I=1,200
      COEF=(1/FACT(I))*(DT**I)*I
      CALL MULT(PRD,DFMTX,PRD2,K1PRD,K2PRD,K1DF,K2DF,K1PRD2,K2PRD2)
      CALL SCALAR(PRD2,COEF,TERM,K1PRD2,K2PRD2,K1TERM,K2TERM)
      CALL SCALAR(DPHIX,1.0,SUM,K1DPH,K2DPH,K1SUM,K2SUM)
      CALL ADD(TERM,DPHIX,DPHIX,K1TERM,K2TERM,K1DPH,K2DPH)
      CALL SUB(DPHIX,SUM,DIFF,K1DPH,K2DPH,K1DIFF,K2DIFF)
      CALL MULT(F,PRD,PRD1,K1F,K1F,K1PRD,K2PRD,K1PRD1,K2PRD1)
      CALL SCALAR(PRD1,1.0,PRD,K1PRD1,K2PRD1,K1PRD,K2PRD)
      DO 50 J=1,K1DIFF
      DO 50 K=1,K2DIFF
      IF(ABS(DIFF(J,K))-ERROR)50,50,100
 50   CONTINUE
      GO TO 200
 100  CONTINUE
      WRITE(6,110)
 110  FORMAT(//,' ',10X,'200 ITERATIONS. NO CONVERGENCE.')
 200  RETURN
      END
C
C
      SUBROUTINE PARF(THETA,DFMTX)
C        SUBROUTINE PARF CALCULATES THE PARTIAL DERIVATIVE OF THE SYSTEM
C        MATRIX WITH RESPECT TO THETA.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION DFMTX(K1DF,K2DF)
      DIMENSION A(3,3)
      A(1,1)=((-2.0)*F22)/(RMASS*TVEL)
      A(1,2)=(2.0*F22)/RMASS
      A(1,3)=0.0
      A(2,1)=0.0
      A(2,2)=(RKSI*TVEL)/(2.0*(THETA**2.0)*F11*(RLE**2.0))
      A(2,3)=0.0
      A(3,1)=0.0
      A(3,2)=0.0
      A(3,3)=0.0
      CALL SCALAR(A,1.0,DFMTX,3,3,K1DF,K2DF)
      RETURN
      END
C
C
      SUBROUTINE PDP(DFMTX,DELTAP,DELTAK)
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NPECR,
     $NRECD,IROSTA,IROEND,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION RI(3,3)
      DIMENSION HTRI(3,3)
      DIMENSION DELTAK(3,3)
      DIMENSION DFMTX(3,3)
      DIMENSION HT(3,3)
      DIMENSION DFMTXT(3,3)
      DIMENSION HP(3,3)
      DIMENSION RIHP(3,3)
      DIMENSION  PRD1(3,3)
      DIMENSION FT(3,3)
      DIMENSION A11(3,3)
      DIMENSION PRD2(3,3)
      DIMENSION PRD3(3,3)
      DIMENSION A21(3,3)
      DIMENSION RIH(3,3)
      DIMENSION HRIH(3,3)
      DIMENSION PRD4(3,3)
      DIMENSION A22(3,3)
      DIMENSION A12(3,3)
      DIMENSION DELTAP(3,3)
      DOUBLE PRECISION PHI(6,6)
      DIMENSION ZDELP(6,6)
      DOUBLE PRECISION RIDP(3,3)
      DOUBLE PRECISION RDP(3,3)
      DOUBLE PRECISION WKAREA(9)
      NAMELIST/DIMEN/M
 01   CALL TRANS(F,FT,K1F,K1F,K1FT,K2FT)
      CALL SPTODP(R,K1R,K1R,RDP)
      IDGT=0
      CALL LINV1F(RDP,K1R,K1R,RIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(RIDP,K1R,K1R,RI)
      CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL TRANS(DFMTX,DFMTXT,K1P,K1P,K1DFM,K2DFM)
      CALL MULT(H,P,HP,K1H,K2H,K1P,K1P,K1HP,K2HP)
      CALL MULT(RI,HP,RIHP,K1R,K1R,K1HP,K2HP,K1RIHP,K2RIHP)
      CALL MULT(HT,RIHP,PRD1,K1HT,K2HT,K1RIHP,K2RIHP,K1PRD1,K2PRD1)
      CALL SUB(PRD1,FT,A11,K1PRD1,K2PRD1,K1A11,K2A11)
      CALL MULT(P,DFMTXT,PRD2,K1P,K1P,K1DFM,K2DFM,K1PRD2,K2PRD2)
      CALL MULT(DFMTX,P,PRD3,K1DFM,K2DFM,K1P,K1P,K1PRD3,K2PRD3)
      CALL ADD(PRD2,PRD3,A21,K1PRD2,K2PRD2,K1A21,K2A21)
      CALL MULT(RI,H,RIH,K1R,K1R,K1H,K2H,K1RIH,K2RIH)
      CALL MULT(HT,RIH,HRIH,K1HT,K2HT,K1RIH,K2RIH,K1HRIH,K2HRIH)
      CALL MULT(P,HRIH,PRD4,K1P,K1P,K1HRIH,K2HRIH,K1PRD4,K2PRD4)
      CALL SUB(F,PRD4,A22,K1F,K1F,K1A22,K2A22)
      CALL SCALAR(F,0.0,A12,K1F,K1F,K1A12,K2A12)
      DO 20 I=1,3
      DO 20 J=1,3
      ZDELP(I,J)=A11(I,J)
 20   CONTINUE
      DO 40 I=1,3
      DO 40 J=1,3
      M=I
      N=J+K2A11
      ZDELP(M,N)=A12(I,J)
 40   CONTINUE
      DO 60 I=1,3
      DO 60 J=1,3
      M=I+K1A11
      N=J
      ZDELP(M,N)=A21(I,J)
 60   CONTINUE
      DO 80 I=1,3
      DO 80 J=1,3
      M=I+K1A11
      N=J+K2A11
      ZDELP(M,N)=A22(I,J)
 80   CONTINUE
      WRITE(6,100)
 100  FORMAT(//,' ',12X,'Z MATRIX FOR SOLUTION TO PARTIAL(P)/',
     $'PARTIAL(THETA)')
      CALL OUTPUT(ZDELP,M,M)
      WRITE(6,DIMEN)
      CALL STMZ(ZDELP,DT,PHI,M,K1PHI,K2PHI)
      WRITE(6,120)
 120  FORMAT(//,' ',12X,'STM FOR SOLUTION TO PARTIAL(P)/',
     $'PARTIAL(THETA)')
      CALL OUTDP(PHI,K1PHI,K2PHI)
      CALL RICATI(PHI,K1PHI,DELTAP,K1DELP)
      CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL MULT(HT,RI,HTRI,K1HT,K2HT,K1R,K1R,K1HTRI,K2HTRI)
      CALL MULT(DELTAP,HTRI,DELTAK,K1DELP,K2DELP,K1HTRI,K2HTRI,K1DELK,
     $K2DELK)
      RETURN
      END
C
C
      SUBROUTINE XSEN(PHIK,K1PHIK,DFMTX,DELTAK,XHAT,ZMEAS,DXHAT)
C        SUBROUTINE XSEN CALCUALTES THE PARTIAL DERIVATIVE OF XHAT
C        WITH RESPECT TO THETA(THE VECTOR OF UNKNOWNS.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
      DOUBLE PRECISION PHIK(3,3)
      DOUBLE PRECISION PHIKI(3,3)
      DOUBLE PRECISION WKAREA(9)
      DOUBLE PRECISION DPKH(3,3)
      DOUBLE PRECISION DPKHI(3,3)
      DIMENSION SPHIKI(3,3)
      DIMENSION DFMTX(3,3)
      DIMENSION DELTAK(3,3)
      DIMENSION XHAT(3,NPOINT)
      DIMENSION ZMEAS(3,NPOINT)
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       DIMENSION DXHAT(3,NPOINT)
       DIMENSION RM(3,3)
       DIMENSION PRD1(3,3)
       DIMENSION XINPT(3,3)
       DIMENSION SPHIK(3,3)
       DIMENSION RKGH(3,3)
       DIMENSION PKH(3,3)
       DIMENSION PRD2(3,3)
       DIMENSION PRD3(3,3)
       DIMENSION PRD4(3,3)
       DIMENSION PRD5(3,3)
       DIMENSION PRD6(3,3)
       DIMENSION PKHI(3,3)
       DIMENSION VXHAT(3,1)
       DIMENSION VDXHAT(3,1)
       DIMENSION VZ(3,1)
       DIMENSION TERM1(3,1)
       DIMENSION TERM2(3,1)
       DIMENSION TERM3(3,1)
       DIMENSION SUM1(3,1)
       DIMENSION VXSEN(3,1)
       DIMENSION GAMMA(3,3)
       DIMENSION LAMBDA(3,3)
       CALL MULT(DELTAK,H,PRD1,K1DELK,K2DELK,K1H,K2H,K1PRD1,K2PRD1)
       CALL SUB(DPMTX,PRD1,XINPT,K1P,K1P,K1XINP,K2XINP)
       CALL DPTOSP(PHIK,3,3,SPHIK)
       FACTOR=0.0
       CALL SCALAR(P,FACTOR,RM,K1P,K1P,K1RM,K2RM)
       DO 40 I=1,K1P
       RM(I,I)=1.0
 40    CONTINUE
       IDGT=0
       CALL LINV1F(PHIK,K1PHIK,K1PHIK,PHIKI,IDGT,WKAREA,IER)
       CALL DPTOSP(PHIKI,K1PHIK,K1PHIK,SPHIKI)
       CALL MULT(RKGM,H,RKGH,K1RKGM,K2RKGM,K1H,K2H,K1RKGH,K2RKGH)
       CALL SUB(P,RKGH,PKH,K1P,K1P,K1PKH,K2PKH)
       CALL SPTODP(PKH,K1PKH,K2PKH,DPKH)
       IDGT=0
       CALL LINV1F(DPKH,K1PKH,K2PKH,DPKHI,IDGT,WKAREA,IER)
       CALL SUB(RM,SPHIKI,PRD2,K1RM,K2RM,K1PRD2,K2PRD2)
       CALL MULT(PRD2,XINPT,PRD3,K1PRD2,K2PRD2,K1XINP,K2XINP,K1PRD3,
      $K2PRD3)
       CALL DPTOSP(DPKHI,K1PKH,K2PKH,PKHI)
       CALL MULT(PKHI,PRD3,PRD4,K1PKH,K2PKH,K1PRD3,K2PRD3,K1PRD4,K2PRD4)
       CALL MULT(SPHIK,PRD4,GAMMA,K1PHIK,K1PHIK,K1PRD4,K2PRD4,K1GAMA,
      $K2GAMA)
       CALL MULT(PRD2,DELTAK,PRD5,K1PRD2,K2PRD2,K1DELK,K2DELK,K1PRD5,
      $K2PRD5)
       CALL MULT(PKHI,PRD5,PRD6,K1PKH,K2PKH,K1PRD5,K2PRD5,K1PRD6,K2PRD6)
       CALL MULT(SPHIK,PRD6,LAMBDA,K1PHIK,K1PHIK,K1PRD6,K2PRD6,
      $K1LAMB,K2LAMB)
       DO 200 J=1,3
       VDXHAT(J,1)=0.0
 200   CONTINUE
       DO 500 I=1,NPOINT
       DO 220 J=1,3
       VXHAT(J,1)=XHAT(J,I)
       VZ(J,1)=ZMEAS(J,I)
 220   CONTINUE
       CALL MULT(SPHIK,VDXHAT,TERM1,K1PHIK,K1PHIK,3,1,K1TER1,K2TER1)
      CALL MULT(GAMMA,VXHAT,TERM2,K1GAMA,K2GAMA,3,1,K1TER2,K2TER2)
      CALL MULT(LAMBDA,VZ,TERM3,K1LAMB,K2LAMB,3,1,K1TER3,K2TER3)
      CALL ADD(TERM1,TERM2,SUM1,K1TER1,K2TER1,K1SUM1,K2SUM1)
      CALL ADD(SUM1,TERM3,VXSEN,K1SUM1,K2SUM1,K1VXSN,K2VXSN)
      DO 400 J=1,3
      DXHAT(J,I)=VXSEN(J,1)
 400  CONTINUE
      CALL SCALAR(VXSEN,1.0,VDXHAT,K1VXSN,K2VXSN,K1VDHT,K2VDHT)
 500  CONTINUE
      RETURN
      END
C
C
      SUBROUTINE GRADE(DXHAT,RNU,DELTAP,DELTAJ)
C        SUBROUTINE GRADE COMPUTES THE GRADIENT OF THE LIKELIHOOD
C        FUNCTION.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),Q(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,P11,P22,RKY,RKSI,RIE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATIEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $RLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DP,K2DP,IFLAG
      DIMENSION DXHAT(3,NPOINT)
      DIMENSION DELTAP(K1DELP,K2DELP)
      DIMENSION RNU(3,NPOINT)
      DIMENSION A1(3,1)
      DIMENSION A1T(1,3)
      DIMENSION  BI(3,3)
      DIMENSION HT(3,3)
      DIMENSION A2(3,3)
      DIMENSION A3(3,3)
      DIMENSION DELTAB(3,3)
      DIMENSION A4(3,3)
      DIMENSION A5(3,3)
      DIMENSION A6(3,1)
      DIMENSION HA6(3,1)
      DIMENSION A7(3,1)
      DIMENSION A8(3,1)
      DIMENSION TERM1(1,1)
      DIMENSION TERM2(1,1)
      DOUBLE PRECISION WKAREA(9)
      DOUBLE PRECISION BDP(3,3)
      DOUBLE PRECISION BIDP(3,3)
      DIMENSION DELTAV(3,1)
 01   SUM=0.0
      IDGT=0
      CALL SPTODP(B,K1B,K1B,BDP)
      CALL LINV1F(BDP,K1B,K1B,BIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(BIDP,K1B,K1B,BI)
      CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL MULT(DELTAP,HT,A2,K1DELP,K2DELP,K1HT,K2HT,K1A2,K2A2)
      CALL MULT(H,A2,DELTAB,K1H,K2H,K1A2,K2A2,K1DELB,K2DELB)
      CALL MULT(DELTAB,BI,A3,K1DELB,K2DELB,K1B,K1B,K1A3,K2A3)
      CALL MULT(BI,A3,A4,K1B,K1B,K1A3,K2A3,K1A4,K2A4)
      CALL MULT(BI,DELTAB,A5,K1B,K1B,K1DELB,K2DELB,K1A5,K2A5)
      CALL TRACE(A5,T3,K1A5)
      DO 100 I=1,NPOINT
      DO 20 J=1,3
      A1(J,1)=RNU(J,I)
      A6(J,1)=DXHAT(J,I)
 20   CONTINUE
      CALL TRANS(A1,A1T,3,1,K1A1T,K2A1T)
      CALL MULT(H,A6,HA6,K1H,K2H,3,1,K1HA6,K2HA6)
      CALL NEG(HA6,DELTAV,K1HA6,K2HA6,K1DELV,K2DELV)
      CALL MULT(BI,DELTAV,A7,K1B,K1B,K1DELV,K2DELV,K1A7,K2A7)
      CALL MULT(A1T,A7,TERM1,K1A1T,K2A1T,K1A7,K2A7,K1TER1,K2TER1)
      CALL MULT(A4,A1,A8,K1A4,K2A4,3,1,K1A8,K2A8)
      CALL MULT(A1T,A8,TERM2,K1A1T,K2A1T,K1A8,K2A8,K1TER2,K2TER2)
      T1=TERM1(1,1)
      T2=TERM2(1,1)
      SUM=SUM+T1+((1.0/2.0)*(T3-T2))
 100  CONTINUE
      DELTAJ=SUM
      RETURN
      END
C
C
      SUBROUTINE FISHER(RNU,DXHAT,DELTAP,DELJ2)
C        SUBROUTINE FISHER COMPUTES THE FISHER INFORMATION MATRIX.
C        THE FISHER INFORMATION MATRIX IS THE SECOND PARTIAL DERIVATIVE
C        OF THE LIKELIHOOD FUNCTION WITH RESPECT TO THETA.
      COMMON F(3,3),G(3,1),H(3,3),R(3,3),C(1,1),K1F,K1G,K2G,K1H,K2H,K1Q,
     $K1R,RKGM(3,3),K1PHIX,K2PHIX,K1PHIZ,K2PHIZ,
     $K1RKGM,K2RKGM,RMASS,F11,F22,RKY,RKSI,RLE,ALPHA,RZERO,DT,NRECR,
     $NRECD,IR0STA,IR0END,IR1STA,IR1END,IDATSA,IDATEN,NDATA,NRCAL,
     $ATTEN(4,3),RVCAL,TVEL,GRCAL,NPOINT,SDT,P(3,3),B(3,3),K1B,K1P,
     $BLIKE,K1DPH,K2DPH,K1DRKG,K2DRKG,NPT1,XIC(3,1),BZERO,K1DELP,
     $K2DELP,K1DELK,K2DELK,K1DF,K2DF,IFLAG
      DIMENSION RNU(3,NPOINT)
      DIMENSION DXHAT(3,NPOINT)
      DIMENSION DELTAP(K1DELP,K2DELP)
      DIMENSION DELTAE(3,3)
      DIMENSION HNEG(3,3)
      DIMENSION HT(3,3)
      DIMENSION DELNU(3,1)
      DIMENSION DELNUT(1,3)
      DIMENSION DXHAT1(3,1)
      DIMENSION RNU1(3,1)
      DIMENSION RN1T(1,3)
      DIMENSION TERM1(1,1)
      DIMENSION TERM2(1,1)
      DIMENSION TERM3(1,1)
      DIMENSION A1(3,3)
      DIMENSION PRD11(3,1)
      DIMENSION PRD21(3,1)
      DIMENSION PRD22(3,1)
      DIMENSION PRD23(3,1)
      DIMENSION PRD31(3,1)
      DIMENSION PRD32(3,1)
      DIMENSION PRD33(3,1)
      DIMENSION PRD41(3,3)
      DIMENSION PRD42(3,3)
      DIMENSION PRD43(3,3)
      DIMENSION BI(3,3)
      DOUBLE PRECISICN WKAREA(9)
      DOUBLE PRECISICN BDP(3,3)
      DOUBLE PRECISICN BIDP(3,3)
 01   SUM=0.0
      IDGT=0
      CALL SPTODP(B,K1B,K1B,BDP)
      CALL LINV1F(BDP,K1B,K1B,BIDP,IDGT,WKAREA,IER)
      CALL DPTOSP(BIDP,K1B,K1B,BI)
      CALL TRANS(H,HT,K1H,K2H,K1HT,K2HT)
      CALL NEG(H,HNEG,K1H,K2H,K1HNEG,K2HNEG)
      CALL MULT(DELTAP,HT,A1,K1DELP,K2DELP,K1HT,K2HT,K1A1,K2A1)
      CALL MULT(H,A1,DELTAB,K1H,K2H,K1A1,K2A1,K1DELB,K2DELB)
      DO 100 I=1,NPOINT
      DO 20 J=1,3
      DXHAT1(J,1)=DXHAT(J,I)
      RNU1(J,1)=RNU(J,I)
   20 CONTINUE
C        COMPUTE THE 1ST TERM OF THE INFORMATION MATRIX
      CALL MULT(HNEG,DXHAT1,DELNU,K1HNEG,K2HNEG,3,1,K1DNU,K2DNU)
      CALL MULT(BI,DELNU,PRD11,K1B,K1B,3,1,K1PD11,K2PD11)
      CALL TRANS(DELNU,DELNUT,K1DNU,K2DNU,K1DNUT,K2DNUT)
      CALL MULT(DELNUT,PRD11,TERM1,K1DNUT,K2DNUT,K1PD11,K2PD11,K1TER1,
     $K2TER1)
C        COMPUTE THE 2ND TERM OF THE INFORMATION MATRIX
      CALL MULT(BI,DELNU,PRD21,K1B,K1B,K1DNU,K2DNU,K1PD21,K2PD21)
      CALL MULT(DELTAB,PRD21,PRD22,K1DELB,K2DELB,K1PD21,K2PD21,K1PD22,
     $K2PD22)
      CALL MULT(BI,PRD22,PRD23,K1B,K1B,K1PD22,K2PD22,K1PD23,K2PD23)
      CALL TRANS(RNU1,RN1T,3,1,K1RN1T,K2RN1T)
      CALL MULT(RN1T,PRD23,TERM2,K1RN1T,K2RN1T,K1PD23,K2PD23,K1TER2,
     $K2TER2)
C        COMPUTE THE 3RD TERM OF THE INFORMATION MATRIX
      CALL MULT(BI,DELNU,PRD31,K1B,K1B,K1DNU,K2DNU,K1PD31,K2PD31)
      CALL MULT(DELTAB,PRD31,PRD32,K1DELB,K2DELB,K1PD31,K2PD31,K1PD32,
     $K2PD32)
      CALL MULT(BI,PRD32,PRD33,K1B,K1B,K1PD32,K2PD32,K1PD33,K2PD33)
      CALL MULT(RN1T,PRD33,TERM3,K1RN1T,K2RN1T,K1PD33,K2PD33,K1TER3,
     $K2TER3)
C        COMPUTE THE 4TH TERM OF THE INFORMATION MATRIX
      CALL MULT(BI,DELTAB,PRD41,K1B,K1B,K1DELB,K2DELB,K1PD41,K2PD41)
      CALL MULT(DELTAB,PRD41,PRD42,K1DELB,K2DELB,K1PD41,K2PD41,K1PD42,
     $K2PD42)
      CALL MULT(BI,PRD42,PRD43,K1B,K1B,K1PD42,K2PD42,K1PD43,K2PD43)
      CALL TRACE(PRD43,T4,K1PD43)
      T1=TERM1(1,1)
      T2=TERM2(1,1)
      T3=TERM3(1,1)
      PSUM=T1-T2-T3-((1.0/2.0)*T4)
      SUM=SUM+PSUM
  100 CONTINUE
      DELJ2=SUM
      RETURN
      END
C
C
      SUBROUTINE STEP(THETA,DELTAJ,DELJ2,DTHETA,THETAN)
C        SUBROUTINE STEP CALCULATES THE NEW VALUE OF THETA (THETAN).
      DTHETA=(-1.0/DELJ2)*DELTAJ
      THETAN=DTHETA+THETA
      RETURN
      END
C
C
      SUBROUTINE MULT(A,B,D,K1A,K2A,K1B,K2B,K1D,K2D)
      DIMENSION A(K1A,K2A),B(K1B,K2B),D(K1A,K2B)
      IF(K2A .NE. K1B)GC TO 40
      DO 20 I=1,K1A
      DO 20 J=1,K2B
      L=0
      DO 20 K=1,K2A
      L=L+1
      IF(L .NE. 1)GO TO 20
      D(I,J)=0
   20 D(I,J)=D(I,J)+(A(I,K)*B(K,J))
      K1D=K1A
      K2D=K2B
      GO TO 80
   40 WRITE(6,60)
   60 FORMAT(1H0,'MATRICES A AND B ARE NOT CCNFORMABLE')
   80 RETURN
      END
C
C
      SUBROUTINE SCALAR(A,B,C,K1A,K2A,K1C,K2C)
      DIMENSION A(K1A,K2A),C(K1A,K2A)
      DO 10 I=1,K1A
      DO 10 J=1,K2A
   10 C(I,J)=B*A(I,J)
      K1C=K1A
      K2C=K2A
      RETURN
      END
C
C
      SUBROUTINE TRANS(A,B,K1A,K2A,K1B,K2B)
      DIMENSION A(K1A,K2A),B(K2A,K1A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
   20 B(J,I)=A(I,J)
      K1B=K2A
      K2B=K1A
      RETURN
      END
C
C
      SUBROUTINE NEG(A,B,K1A,K2A,K1B,K2B)
      DIMENSION A(K1A,K2A),B(K1A,K2A)
      DO 10 I=1,K1A
      DO 10 J=1,K2A
   10 B(I,J)=0-A(I,J)
      K1B=K1A
      K2B=K2A
      RETURN
      END
C
C
      SUBROUTINE ADD(A,B,C,K1A,K2A,K1C,K2C)
      DIMENSION A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
   20 C(I,J)=A(I,J)+B(I,J)
      K1C=K1A
      K2C=K2A
      RETURN
      END
```

```
C
C
      SUBROUTINE SUB(A,B,C,K1A,K2A,K1C,K2C)
      DIMENSION A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
  20  C(I,J)=A(I,J)-B(I,J)
      K1C=K1A
      K2C=K2A
      RETURN
      END
C
C
      FUNCTION FACT(K)
      FACT=1.0
      DO 20 I=1,K
      RI=I
  20  FACT=FACT*RI
      RETURN
      END
C
C
      SUBROUTINE TRACE(A,B,K1A)
      DIMENSION A(K1A,K1A)
      SUM=0.0
      DO 20 I=1,K1A
      SUM=SUM+A(I,I)
  20  CONTINUE
      B=SUM
      RETURN
      END
C
C
      SUBROUTINE OUTPUT(A,K1A,K2A)
      DIMENSION A(K1A,K2A)
      DO 20 I=1,K1A
      WRITE(6,30) (A(I,J),J=1,K2A)
  20  CONTINUE
  30  FORMAT(' ',12X,6(2X,E15.8))
      RETURN
      END
C
C
      SUBROUTINE OVER(RMATRX,CHK,K1,K2,IFLAGO)
      DIMENSION RMATRX(K1,K2)
      DO 20 I=1,K1
      DO 20 J=1,K2
      IF(RMATRX(I,J) .GE. CHK)IFLAGO=1
  20  CONTINUE
      RETURN
      END
C
      SUBROUTINE SETCDP(X,K1X,K2X,Y)
      DIMENSION X(K1X,K2X)
      DOUBLE PRECISION Y(K1X,K2X)
      DO 20 I=1,K1X
      DO 20 J=1,K2X
      Y(I,J)=DBLE(X(I,J))
  20  CONTINUE
      RETURN
      END
C
C
      SUBROUTINE DPTOSP(X,K1X,K2X,Y)
      DOUBLE PRECISION X(K1X,K2X)
      DIMENSION Y(K1X,K2X)
      DO 20 I=1,K1X
      DO 20 J=1,K2X
      Y(I,J)=X(I,J)
   20 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE SCALDP(A,B,C,K1A,K2A,K1C,K2C)
      DOUBLE PRECISION A(K1A,K2A),C(K1A,K2A)
      DOUBLE PRECISION B
      DO 10 I=1,K1A
      DO 10 J=1,K2A
   10 C(I,J)=B*A(I,J)
      K1C=K1A
      K2C=K2A
      RETURN
      END
C
C
C
      SUBROUTINE TRANSD(A,B,K1A,K2A,K1B,K2B)
      DOUBLE PRECISION  A(K1A,K2A),B(K2A,K1A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
   20 B(J,I)=A(I,J)
      K1B=K2A
      K2B=K1A
      RETURN
      END
C
C
      SUBROUTINE NEGDP(A,B,K1A,K2A,K1B,K2B)
      DOUBLE PRECISION A(K1A,K2A),B(K1A,K2A)
      DO 10 I=1,K1A
      DO 10 J=1,K2A
   10 B(I,J)=DBLE(0.0)-A(I,J)
      K1B=K1A
      K2B=K2A
      RETURN
      END
C
C
      SUBROUTINE ADDDP(A,B,C,K1A,K2A,K1C,K2C)
      DOUBLE PRECISION A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
   20 C(I,J)=A(I,J)+B(I,J)
      K1C=K1A
      K2C=K2A
      RETURN
      END
C
C
```

```
      SUBROUTINE MULTDP(A,B,D,K1A,K2A,K1B,K2B,K1D,K2D)
      DOUBLE PRECISION A(K1A,K2A),B(K1B,K2B),D(K1A,K2B)
      IF(K2A .NE. K1B)GO TO 40
      DO 20 I=1,K1A
      DO 20 J=1,K2B
      L=0
      DO 20 K=1,K2A
      L=L+1
      IF(L .NE. 1)GO TO 20
      D(I,J)=DBLE(0.0)
   20 D(I,J)=D(I,J)+(A(I,K)*B(K,J))
      K1D=K1A
      K2D=K2B
      GO TO 80
   40 WRITE(6,60)
   60 FORMAT(1H0,'MATRICES A AND B ARE NOT CONFORMABLE')
   80 RETURN
      END
C
C
      SUBROUTINE SUBDP(A,B,C,K1A,K2A,K1C,K2C)
      DOUBLE PRECISION A(K1A,K2A),B(K1A,K2A),C(K1A,K2A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
   20 C(I,J)=A(I,J)-B(I,J)
      K1C=K1A
      K2C=K2A
      RETURN
      END
C
C
      SUBROUTINE TRACED(A,B,K1A)
      DOUBLE PRECISION A(K1A,K1A)
      DOUBLE PRECISION SUM,B
      SUM=DBLE(0.0)
      DO 20 I=1,K1A
      SUM=SUM+A(I,I)
   20 CONTINUE
      B=SUM
      RETURN
      END
C
C
C
C
      SUBROUTINE OUTDBL(A,K1A,K2A)
      DOUBLE PRECISION A(K1A,K2A)
      DO 20 I=1,K1A
      DO 20 J=1,K2A
      WRITE(6,30) A(I,J)
   20 CONTINUE
   30 FORMAT(' ',12X,D15.8)
      RETURN
      END
C
C
      SUBROUTINE OVERDP(RMATRX,CHK,K1,K2,IFLAGO)
      DOUBLE PRECISION RMATRX(K1,K2)
      DOUBLE PRECISION CHK
      DO 20 I=1,K1
      DO 20 J=1,K2
      IF(RMATRX(I,J) .GE. CHK)IFLAGO=1
   20 CONTINUE
      RETURN
      END
C
C
      DOUBLE PRECISION FUNCTION FACTDP(K)
      DOUBLE PRECISION DREALI
      FACTDP=DBLE(1.0)
      DO 20 I=1,K
      REALI=I
      DREALI=DBLE(REALI)
   20 FACTDP=FACTDP*DREALI
      RETURN
      END
C
C
      SUBROUTINE OUTDP(A,K1A,K2A)
      DOUBLE PRECISION A(K1A,K2A)
      DO 20 I=1,K1A
      WRITE(6,30) (A(I,J),J=1,K2A)
   20 CONTINUE
   30 FORMAT(' ',12X,6(2X,D15.8))
      RETURN
      END
C
C
      SUBROUTINE PLOT(X,Y,IPOINT)
      DIMENSION X(IPOINT),Y(IPOINT)
      WRITE(6,20)
   20 FORMAT(/,' ',40X,'STATE ESTIMATES')
      CALL WPLOT1(X,Y,IPOINT,4,XHAT)
      RETURN
      END
C
C
      SUBROUTINE VMNMX(X,KX,RMIN,RMAX)
      DIMENSION X(KX)
      RMIN=0.0
      RMAX=0.0
      DO 100 I=1,KX
      IF(X(I) .GT. RMAX)RMAX=X(I)
      IF(X(I) .LT. RMIN)RMIN=X(I)
  100 CONTINUE
      RETURN
      END
C
C
      SUBROUTINE PLXHAT(Y1,Y2,K1Y1,K2Y1,K1Y2,K2Y2,XVECT1,YVECT1,
     $YVECT2,DT,ITIT1,JLAB1)
      DIMENSION ITIT1(10)
      DIMENSION JLAB1(10)
      DIMENSION Y1(K1Y1,K2Y1)
      DIMENSION Y2(K1Y2,K2Y2)
      DIMENSION XVECT1(K2Y2)
      DIMENSION YVECT1(K2Y2)
      DIMENSION YVECT2(K2Y2)
      DATA ICHARA,ICHARE/'A','E'/
      DO 20 I=1,K2Y2
      XVECT1(I)=DT*I
 20   CONTINUE
      DO 200 J=1,K1Y2
      DO 50 I=1,K2Y2
      YVECT1(I)=Y1(J,I)
      YVECT2(I)=Y2(J,I)
 50   CONTINUE
      CALL VMNMX(YVECT1,K2Y2,Y1MIN,Y1MAX)
      CALL VMNMX(YVECT2,K2Y2,Y2MIN,Y2MAX)
      IF(Y1MIN .LT. Y2MIN)YMIN=Y1MIN
      IF(Y2MIN .LT. Y1MIN)YMIN=Y2MIN
      IF(Y2MIN .EQ. Y1MIN)YMIN=Y2MIN
      IF(Y1MAX .GT. Y2MAX)YMAX=Y1MAX
      IF(Y2MAX .GT. Y1MAX)YMAX=Y2MAX
      IF(Y2MAX .EQ. Y1MAX)YMAX=Y2MAX
      CALL WPLOT2(XVECT1(K2Y2),XVECT1(1),YMAX,YMIN)
      CALL WPLOT3(ICHARA,XVECT1,YVECT1,K2Y2)
      CALL WPLOT3(ICHARB,XVECT1,YVECT2,K2Y2)
      WRITE(6,90)ITIT1
 90   FORMAT('1',40X,10A4)
      CALL WPLOT4(40,JLAB1)
 200  CONTINUE
      RETURN
      END
C
C
      SUBROUTINE PLXSEN(DXHAT,K1X,K2X,XVECT1,YVECT1,DT,ITIT2,JLAB2)
      DIMENSION DXHAT(K1X,K2X)
      DIMENSION  XVECT1(K2X)
      DIMENSION YVECT1(K2X)
      DIMENSION ITIT2(10)
      DIMENSION JLAB2(10)
      DO 20 I=1,K2X
      XVECT1(I)=DT*I
 20   CONTINUE
      DO 100 J=1,K1X
      DO 50 I=1,K2X
      YVECT1(I)=DXHAT(J,I)
 50   CONTINUE
      WRITE(6,60)ITIT2
 60   FORMAT('1',40X,10A4)
      CALL WPLOT1(XVECT1,YVECT1,K2X,40,JLAB2)
 100  CONTINUE
      RETURN
      END
C
C
      SUBROUTINE CPLCT(X,K1X,K2X,XVECT1,YVECT1,YMIN,YMAX,DT,JLABEL,
     $ITITLE)
      DIMENSION X(K1X,K2X)
      DIMENSION YVECT1(K2X)
      DIMENSION XVECT1(K2X)
      DIMENSION JLABEL(10)
      DIMENSION ITITLE(10)
      DATA ICHAR/'*'/
      DO 20 I=1,K2X
      XVECT1(I)=DT*I
 20   CONTINUE
      DO 200 J=1,K1X
      DO 40 I=1,K2X
      YVECT1(I)=X(J,I)
   40 CONTINUE
      WRITE(6,100)ITITLE
  100 FORMAT('1',40X,10A4)
      CALL WPLOT2(XVECT1(K2X),XVECT1(1),YMAX,YMIN)
      CALL WPLOT3(ICHAR,XVECT1,YVECT1,K2X)
      CALL WPLOT4(40,JLABEL)
  200 CONTINUE
      RETURN
      END
C
```

Appendix F

LIST OF APL FUNCTIONS USED TO GENERATE WHEELSET SIMULATED DATA

```
      ∇ WHLSIM
[1]   A←SYSMTX
[2]   B←INPMTX
[3]   C← 3 3 ⍴0
[4]   D← 3 1 ⍴0
[5]   H← 3 3 ⍴ 1 0 0 0 1 0 0 0 1
[6]   'RANDOM INPUT STATISTICS'
[7]   ⎕←'ENTER SUM LENGTH MEAN MNSQVALUE'
[8]   STATRI←⎕
[9]   W←WHTNOISE STATRI
[10]  'DETERMINISTIC INPUT STATISTICS'
[11]  ⎕←'ENTER SUM LENGTH MEAN MNSQVALUE'
[12]  STATDI←⎕
[13]  WT←WHTNOISE STATDI
[14]  U←(STATRI[2],1,1)⍴W+WT
[15]  DYN
[16]  XMEAS←X
[17]  'MEASUREMENT NOISE STATISTICS'
[18]  ⎕←'ENTER SUM LENGTH MEAN MNSQVALUE'
[19]  STATMN←⎕
[20]  V1←WHTNOISE STATMN
[21]  V2←WHTNOISE STATMN
[22]  V3←WHTNOISE STATMN
[23]  V←(3,STATMN[2])⍴V1,V2,V3
[24]  Z←(H+.×XMEAS)+V
      ∇
      .


      ∇ F←SYSMTX
[1]   F11←¯1×(((2×THETA×COEF22)+(R0×VEL))÷MASS×VEL)
[2]   F12←2×COEF22×THETA÷MASS
[3]   F13←¯1×KT÷MASS
[4]   F21←0
[5]   F22←¯1×KSI×VEL÷2×THETA×COEF11×L×L
[6]   F23←¯1×ALPHA×VEL÷L×R0
[7]   F31←1
[8]   F32←0
[9]   F33←0
[10]  F← 3 3 ⍴F11,F12,F13,F21,F22,F23,F31,F32,F33
      ∇
      .


      ∇ G←INPMTX
[1]   G11←R0÷MASS
[2]   G21←0
[3]   G31←¯1
[4]   G← 3 1 ⍴G11,G21,G31
      ∇
      .
      ∇ Y←WHTNOISE X;A;B;MN;MS
[1]     A←?(X[1]×X[2])⍴100000
[2]     A←(X[1],X[2])⍴A
[3]     B←(1,X[2])⍴+⌿A
[4]     MN←(+/B)÷(×/⍴B)
[5]     B←(¯1×(MN-X[3]))+B
[6]     MS←MNSQVAL B
[7]     B←((X[4]÷MS)⋆0.5)×B
[8]     Y←B
      ∇
        •
```

```
      ∇ Y←MNSQVAL X
[1]     Y←(+/(X+.×⍉X))÷(×/(⍴X))
      ∇
        •
```

```
      ∇ Y←COV X;MN
[1]     MN←(⌽⍴X)⍴(+/X)÷¯1↑⍴X
[2]     MN←⍉MN
[3]     Y←((X-MN)+.×⍉(X-MN))÷¯1↑⍴X
      ∇
        •
```

```
      ∇ DTN
[1]     T←N⍴DT×⍳N
[2]     INITIALIZE
[3]     PHI←DT STM A
[4]     DMATRIX
[5]     M←0
[6]    L0:M←M+1
[7]     XX[;;(M+1)]←(PHI+.×XX[;;M])+DD+.×UU[;;M]
[8]     YY[;;(M+1)]←(C+.×XX[;;(M+1)])+D+.×UU[;;(M+1)]
[9]     →(M<N-1)/L0
[10]    X←(NN,N)⍴,XX
[11]    Y←(LL,N)⍴,YY
      ∇
        •
        ∇ INITIALIZE
[1]     NN←(ρ,A)*0.5
[2]     MM←⌈ρB[1;]
[3]     LL←⌈ρC[;1]
[4]     XX←(NN,1,N)ρ0
[5]     YY←(LL,1,N)ρ0
[6]     UU←(MM,1,N)ρ,U
[7]     'ENTER X̲(0)'
[8]     XX[;1;1]←⎕
[9]     YY[;1;1]←C+.×XX[;1;1]
        ∇
        •
```

```
        ∇ PHI←DT STM A;R;N;FACTOR;INDEX
[1]     R←(ρ,A)*0.5
[2]     I←(⍳R)∘.=⍳R
[3]     N←0
[4]     PHI←I
[5]     FACTOR←I
[6]    L0:N←N+1
[7]     FACTOR←FACTOR+.×A×DT÷N
[8]     PHI←PHI+FACTOR
[9]     INDEX←(+/|,FACTOR)÷R*2
[10]    →(INDEX>EPS)/L0
        ∇
        •
```

```
        ∇ DMATRIX;H
[1]     H←DT÷4
[2]     DD←(H÷3)×((0 STM A)+(4×H STM A)+(2×(2×H) STM A)+(4×(3×H) STM A)+(DT STM A))+.×B
        ∇
        •
```

Appendix G

LIST OF THE PROGRAM USED TO OPERATE THE A/D CONVERTER

```
PAGE 0001  FTN.   9:19 AM  FRI., 13  OCT., 1978

FTN4,L
      PROGRAM DMAD3(3,40)
C     PROGRAM TO DIGITIZE WATER CHANNEL HOT WIRE DATA.
C     THE PROGRAM READS NCH CHANNELS, NPPC SAMPLES PER CHANNEL FROM THE
C     GMAD/1 A/D CONVERTER. THE DATA READ IS WRITTEN ON TAPE.
C     VERSION OF 18 JULY 1978
      DIMENSION IBUFA(512),IBUFB(512)
      INTEGER PARMS(5)
      EQUIVALENCE (PARMS(1),LUCRT),(PARMS(2),IPRINT),(PARMS(3),MAG),
     1            (PARMS(4),LU)
C     GET LOGICAL UNITS
      CALL RMPAR(PARMS)
C     DEFAULT THEM IF NECESSARY
      IF(LUCRT .LE. 0) LUCRT=1
      IF(MAG .LE. 0) MAG=8
      IF(LU .LE. 0) LU=17
      N=0
      NCH=2
      NPPC=256
 2222 FORMAT("ENTER THE NUMBER RECORDS PER FILE <")
      ICLAS=0
C     AT PRESENT S.A.M.(2048 WORDS), IBUFL MUST EQUAL 512 OR LESS
      IBUFL=NCH*NPPC
C     GET A CLASS NUMBER
      CALL EXEC(18,0,IBUFB,IBUFL,0,0,ICLAS)
C     TELL USER WHAT CLASS NUMBER WE GOT
      NUMB=IAND(ICLAS,17777B)
      WRITE(LUCRT,1203) NUMB
 1203 FORMAT("CLASS NUMBER ",@6,"B")
C     SET BIT 15 IN CLASS WORD FOR NO WAIT AND BIT 13 TO KEEP CLASS NR.
      ICLAS=IOR(ICLAS,120000B)
      WRITE(LUCRT,1201)
 1201 FORMAT("ENTER DESIRED SAMPLING RATE IN HZ. <")
      READ(LUCRT,*) HERTZ
C     COMPUTE CLOCK DIVISOR
      ICDV=IFIX(1.0E+07/HERTZ)
      HERTZ=1.0E+07/FLOAT(ICDV)
      WRITE(LUCRT,1202) HERTZ
 1202 FORMAT("ACTUAL SAMPLING RATE IS ",G10.4," HERTZ")
C     CONSTRUCT CONTROL WORD FOR SETTING CLOCK DIVIDER
      ICLK=LU+3000B
C     SET CLOCK DIVIDER
      CALL EXEC(3,ICLK,ICDV)
   15 CONTINUE
      WRITE(LUCRT,2222)
      READ(LUCRT,*) MAXREC
      IF(MAXREC .LE. 0) GO TO 30
C     MAXREC IS DIVIDED BY 2 BECAUSE TWO RECORDS
```

```
C     ARE WRITTEN EACH TIME THROUGH THE LOOP
      MAXREC=MAXREC/2
      DO 25 J=1,MAXREC
C     READ FROM A/D INTO BUFA AND SUSPEND PROGRAM
      CALL EXEC(1,LU,IBUFA,IBUFL)
C     DO CLASS GET. INITIALLY IT WILL (I HOPE) CLEAR THE CLASS REQUEST
C     THAT GOT US THE CLASS NUMBER. AFTERWARDS IT SHOULD CLEAR ONE OF
C     THE CLASS WRITES TO TAPE.
      CALL EXEC(21,ICLAS,IBUFB,IBUFL)
C     BEGIN WRITING BUFA TO MAG TAPE. THEN
C     SIMULTANEOUSLY READ FROM A/D TO BUFB
      CALL EXEC(18,MAG,IBUFA,IBUFL,0,0,ICLAS)
      CALL ABREG(IRETRN,IB)
      IF(IRETRN .NE. 0) WRITE(LUCRT,1204) IRETRN
 1204 FORMAT("CLASS WRITE ERROR ",I2)
      CALL EXEC(1,LU,IBUFB,IBUFL)
C     CHECK IBUFA WRITE FOR COMPLETION
      CALL EXEC(21,ICLAS,IBUFA,IBUFL)
C     START WRITING IBUFB TO TAPE
      CALL EXEC(18,MAG,IBUFB,IBUFL,0,0,ICLAS)
      CALL ABREG(IPETRN,IB)
      IF(IRETRN .NE. 0) WRITE(LUCRT,1204) IRETRN
   25 CONTINUE
      ENDFILE MAG
      N=N+1
      WRITE(LUCRT,1003)N
 1003 FORMAT("FILE NUMBER ",I4," HAS BEEN READ")
      GO TO 15
   30 CONTINUE
C     CLEAR BIT 13 IN CLASS WORD. WE ARE DONE WITH THE CLASS NUMBER.
      ICLAS=IAND(ICLAS,157777B)
C     NOW DO GET TO CLEAR FINAL REQUEST AND RELEASE CLASS NUMBER.
   40 CALL EXEC(21,ICLAS,IBUFB,IBUFL)
      CALL ABREG(IRETRN,IB)
      WRITE(LUCRT,1205) IRETRN
 1205 FORMAT("FINAL GET RETURNED ",@6)
      IF(IRETRN .LT. 0) GO TO 40
      ENDFILE MAG
      STOP
      END


 FTN4 COMPILER: HP92060-16092 REV. 1805 (780310)


 **  NO WARNINGS **  NO ERRORS **   PROGRAM = 01450      COMMON = 00000
```

Appendix H

PLOTS OF THE LIKELIHOOD FUNCTION, OBSERVATION TERM AND BIAS TERM FOR TEST CASES 1 THROUGH 9

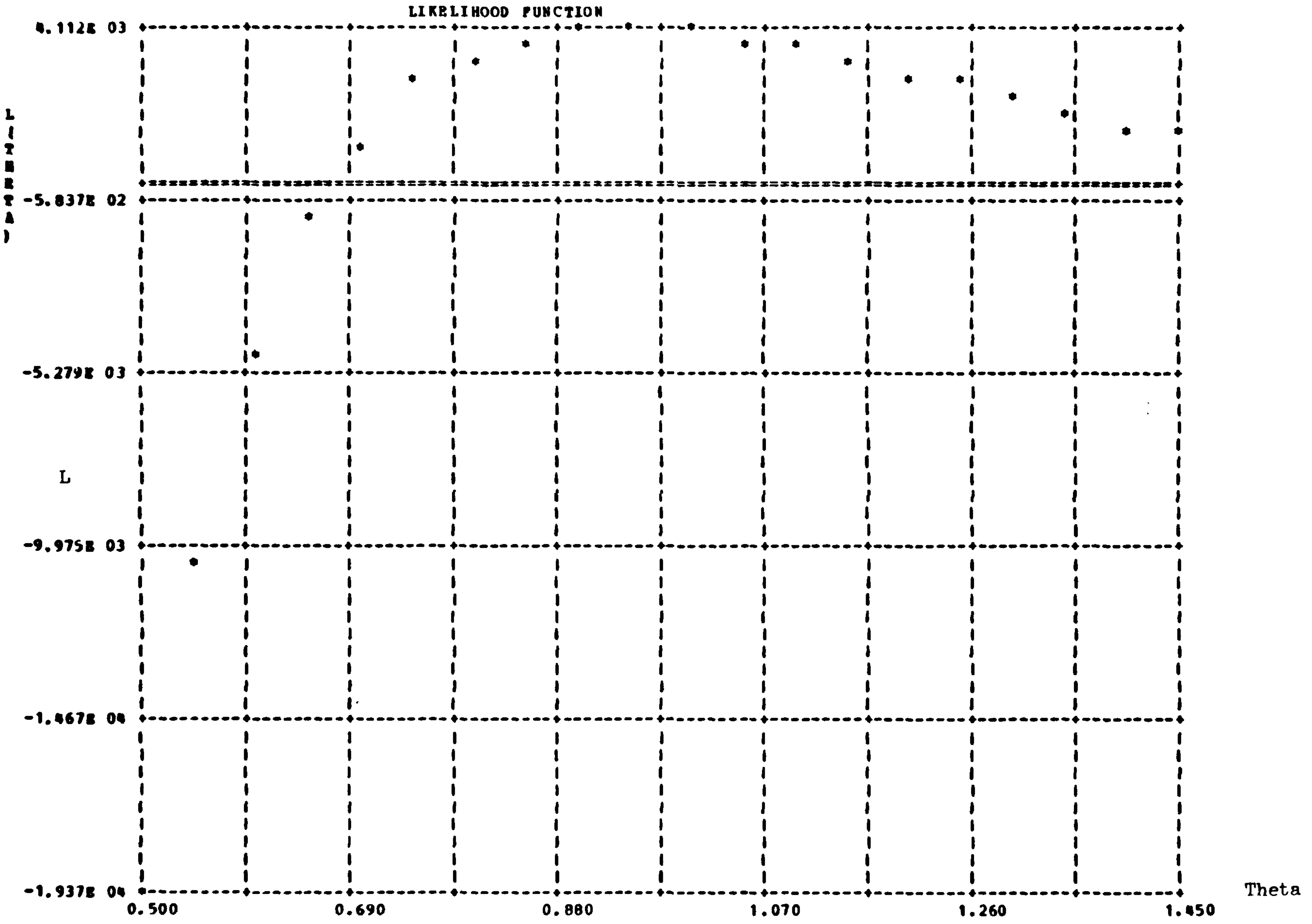

Figure H.1 Plot of the likelihood function for test case 1.

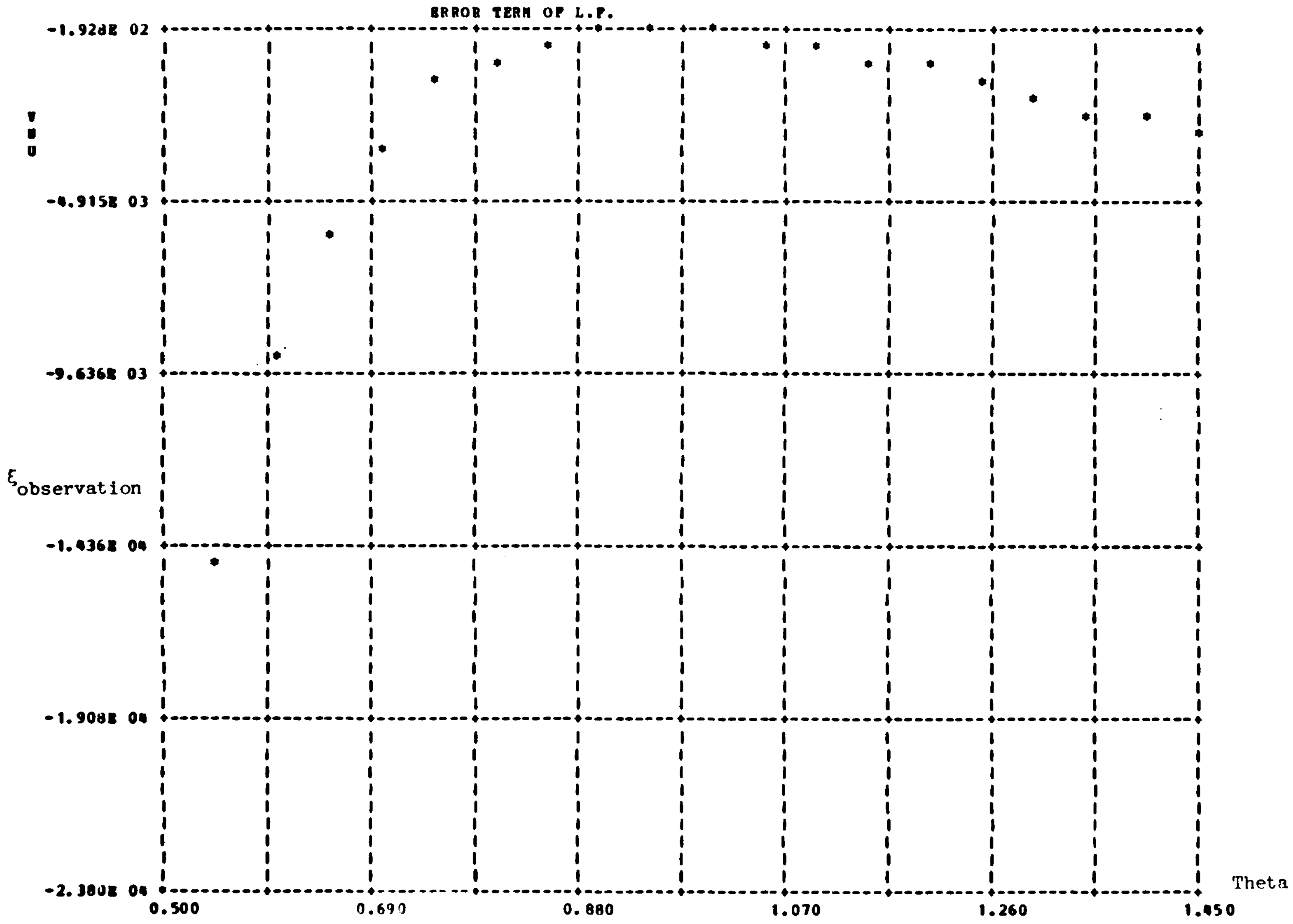

Figure II.2 Plot of the observation term for test case 1.

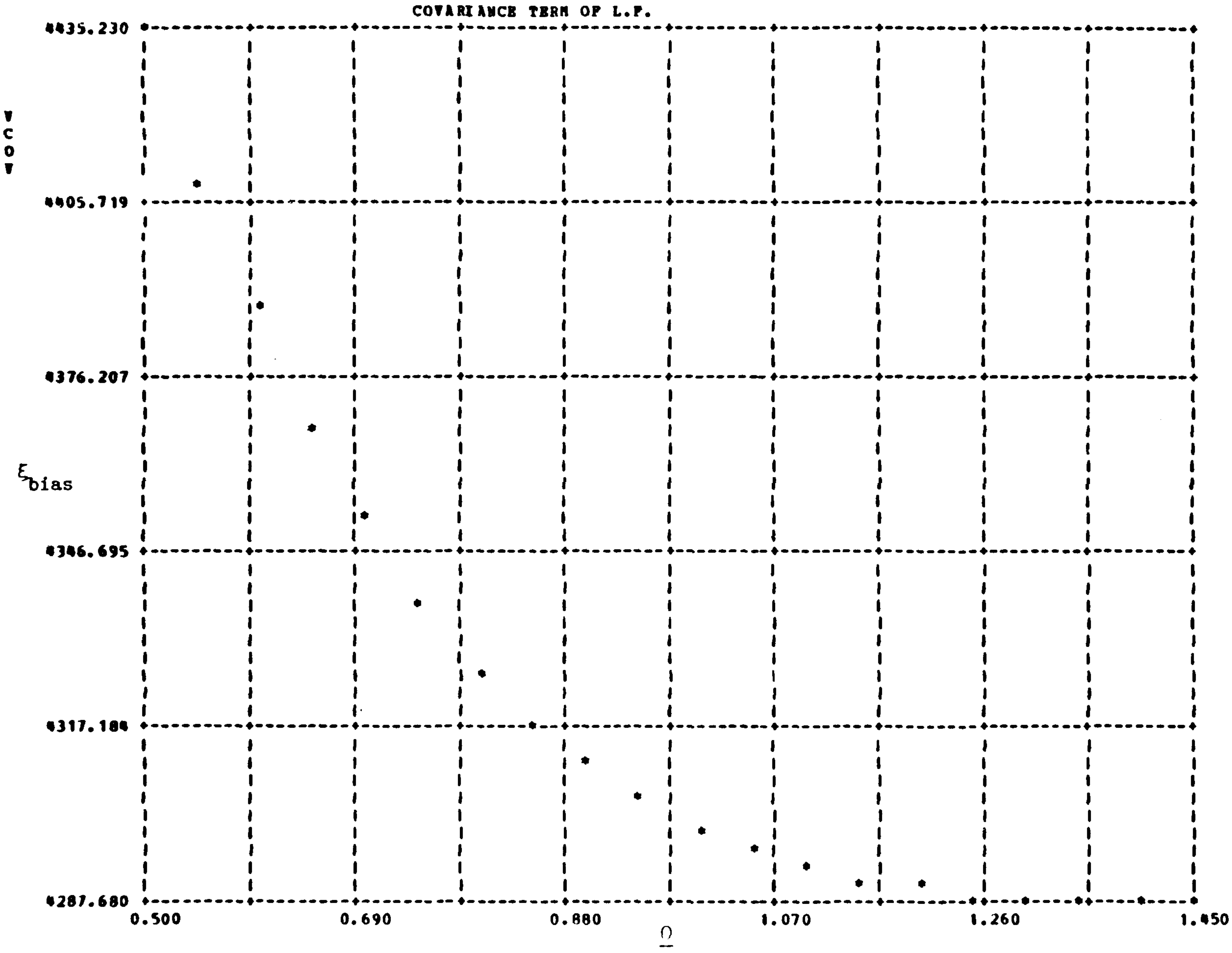

Figure H.3 Plot of the bias term for test case 1.

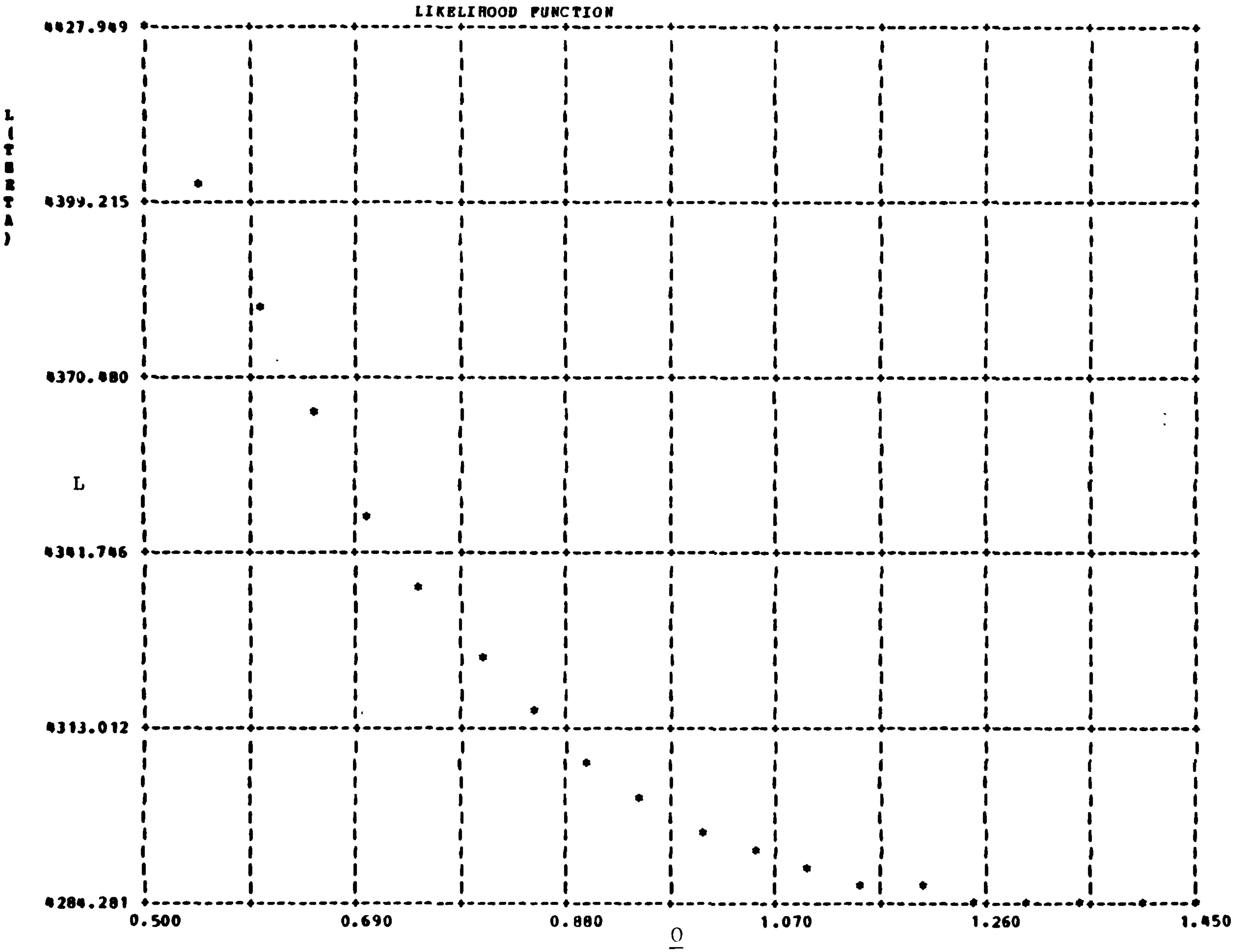

Figure H.4 Plot of the likelihood function for test case 2.

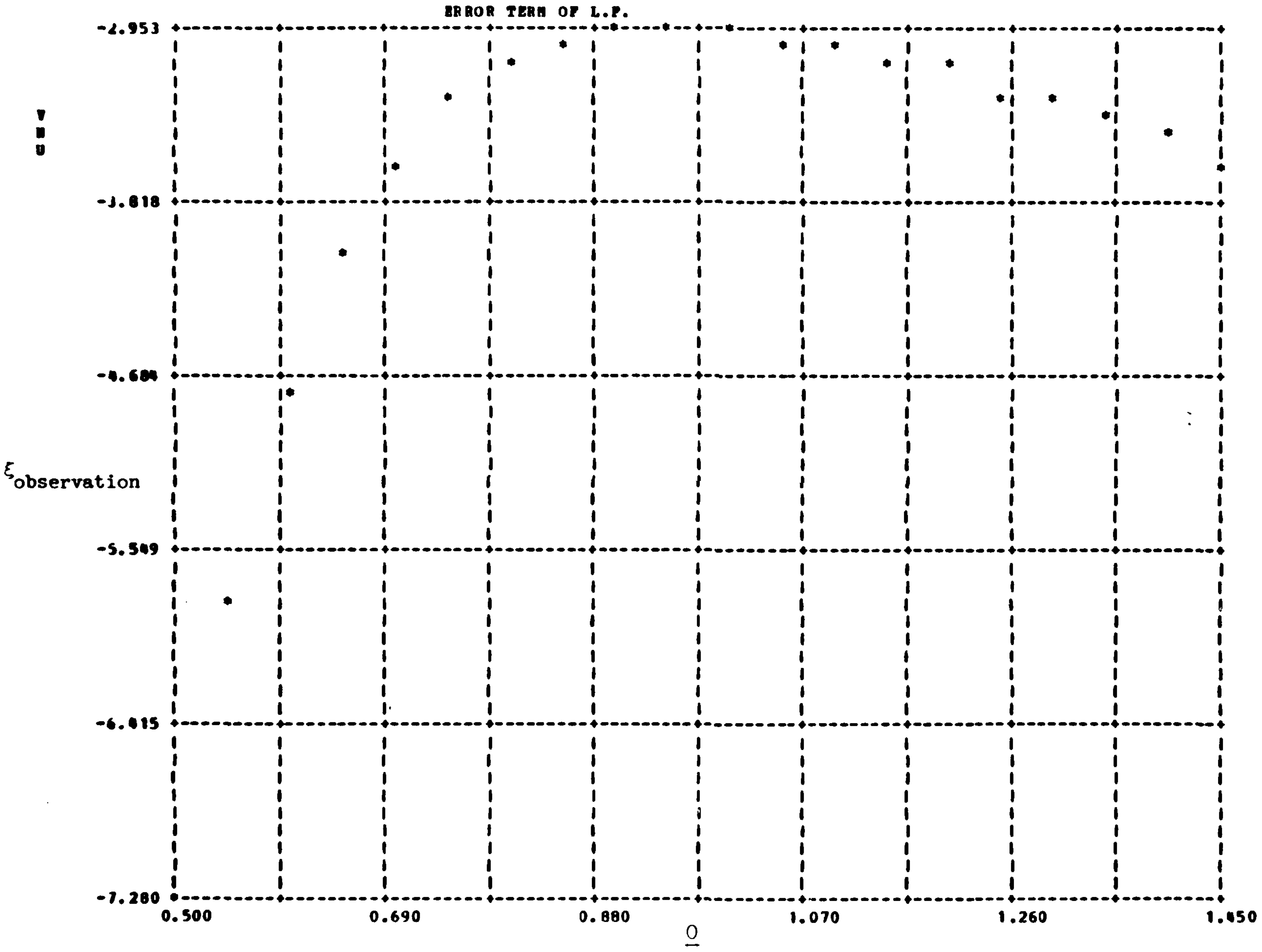

Figure H.5 Plot of the observation term for test case 2.

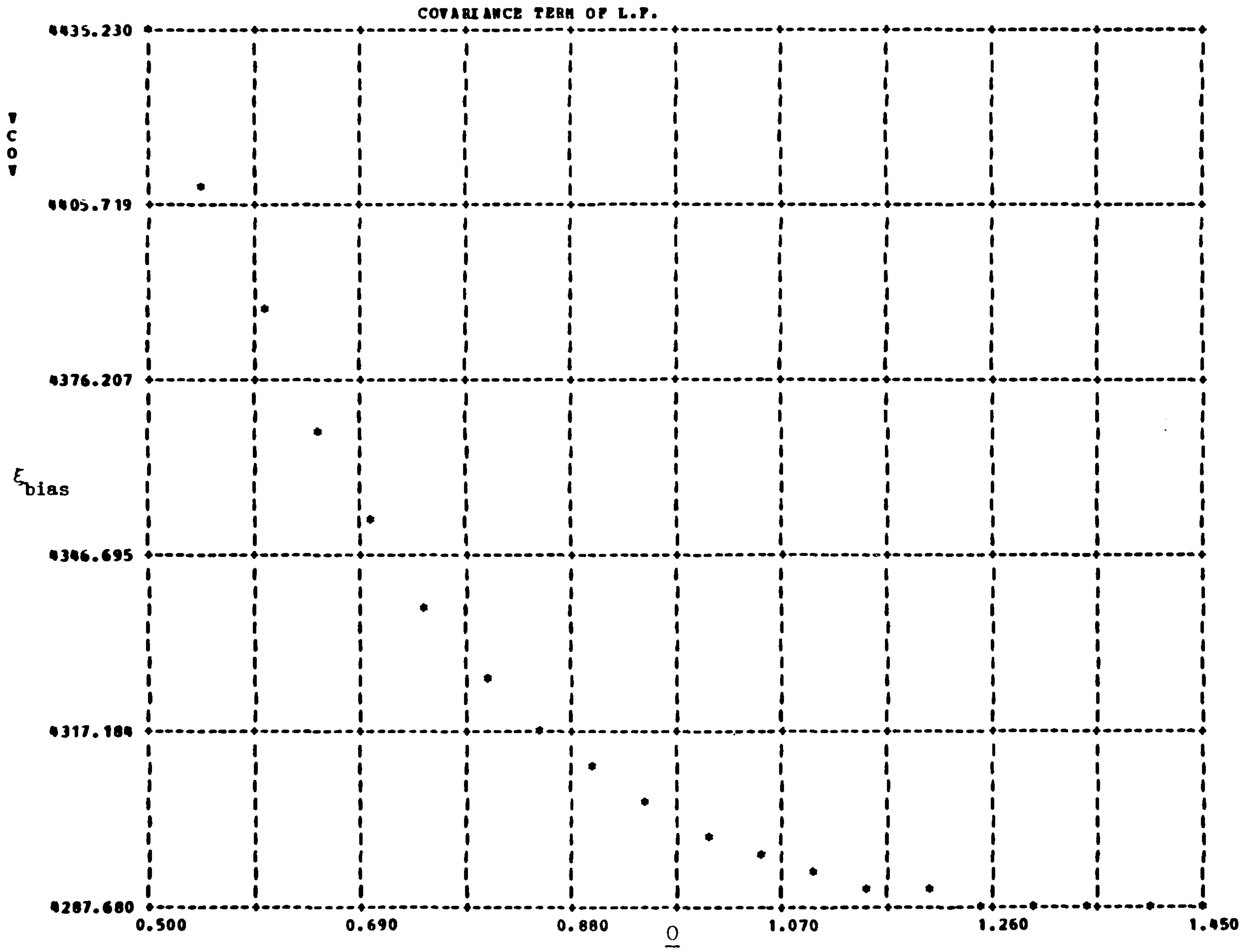

Figure H.6 Plot of the bias term for test case 2.

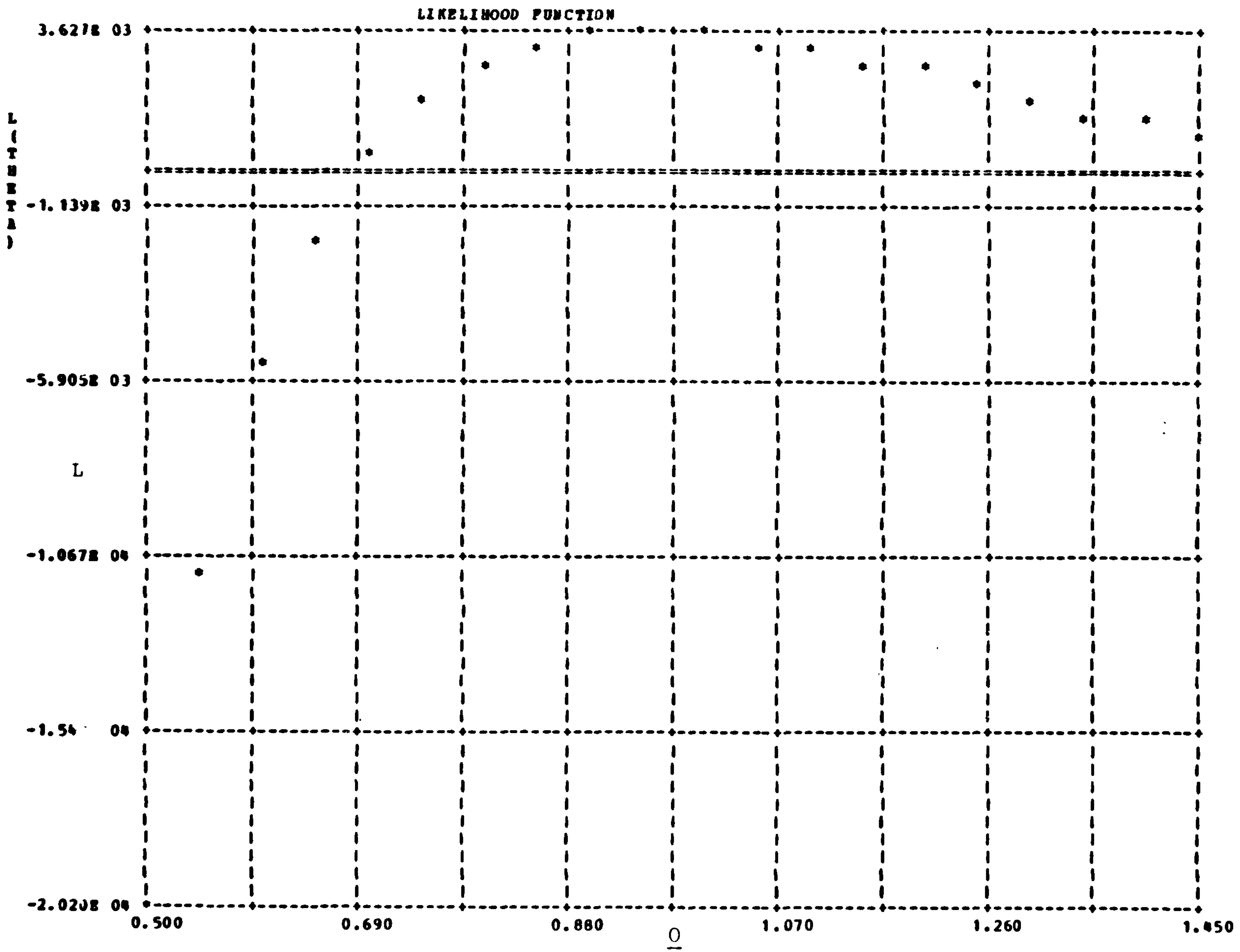

Figure H.7 Plot of the likelihood function for test case 3.

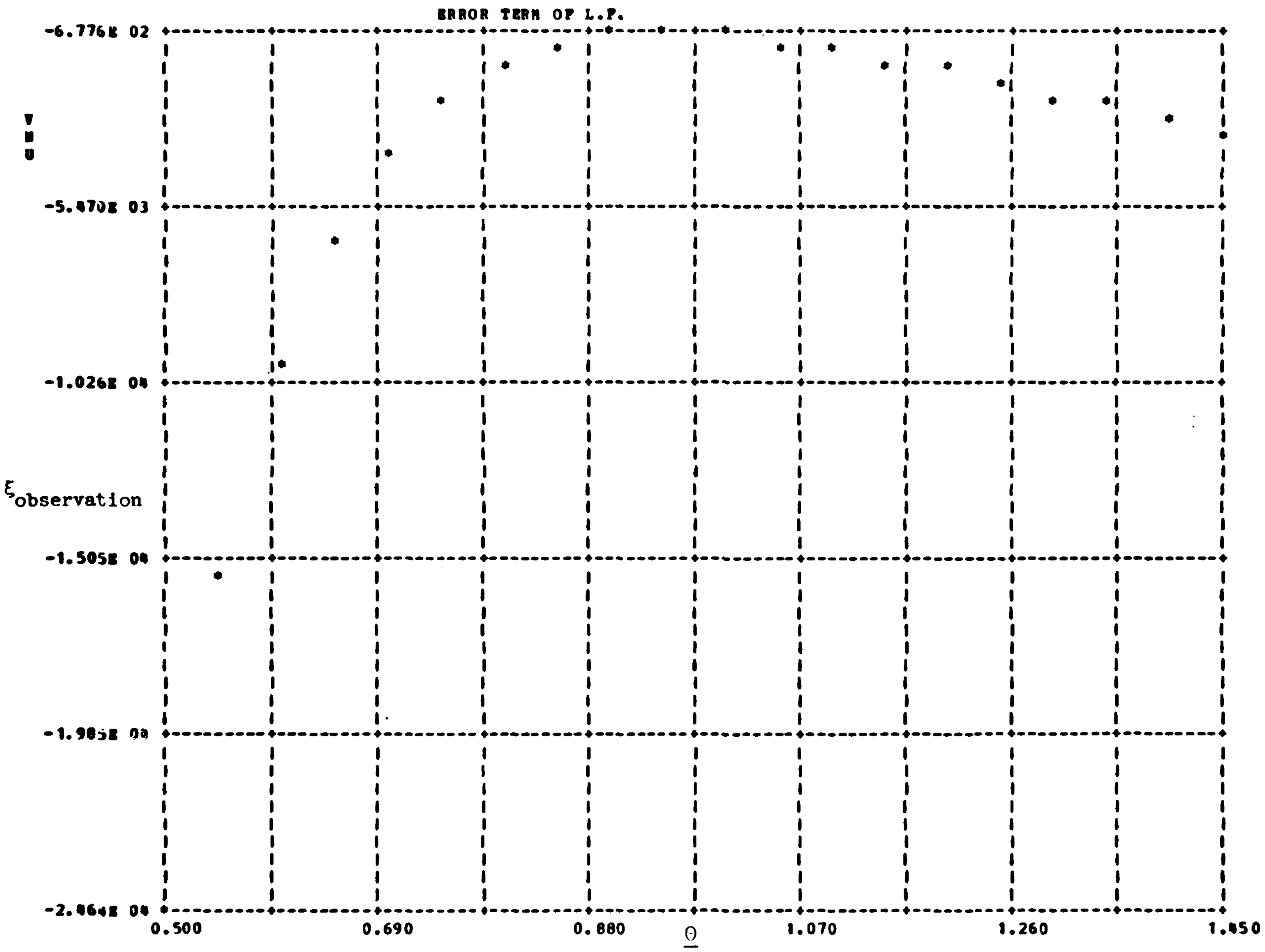

Figure H.8 Plot of the observation term for test case 3.

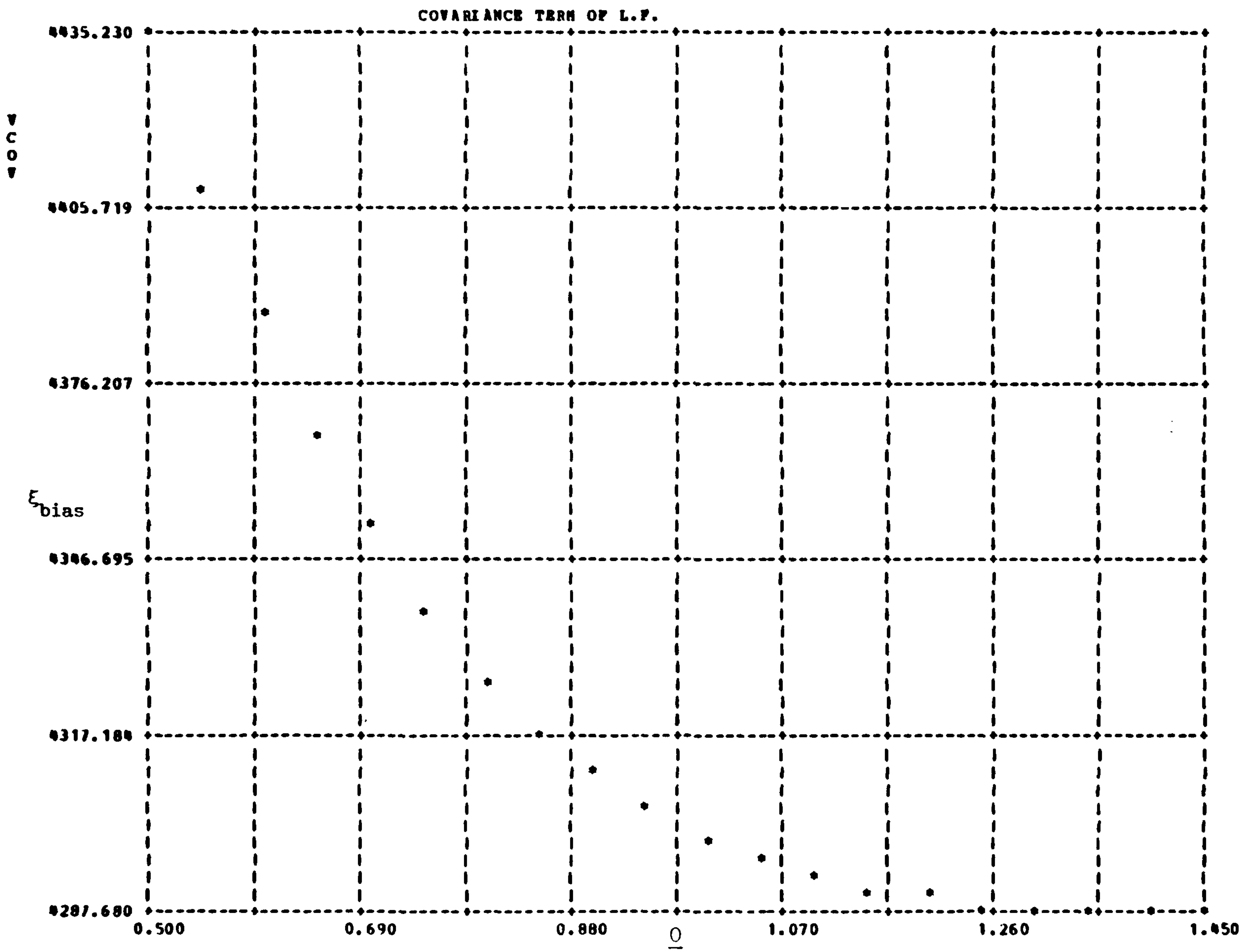

Figure H.9 Plot of the bias term for test case 3.

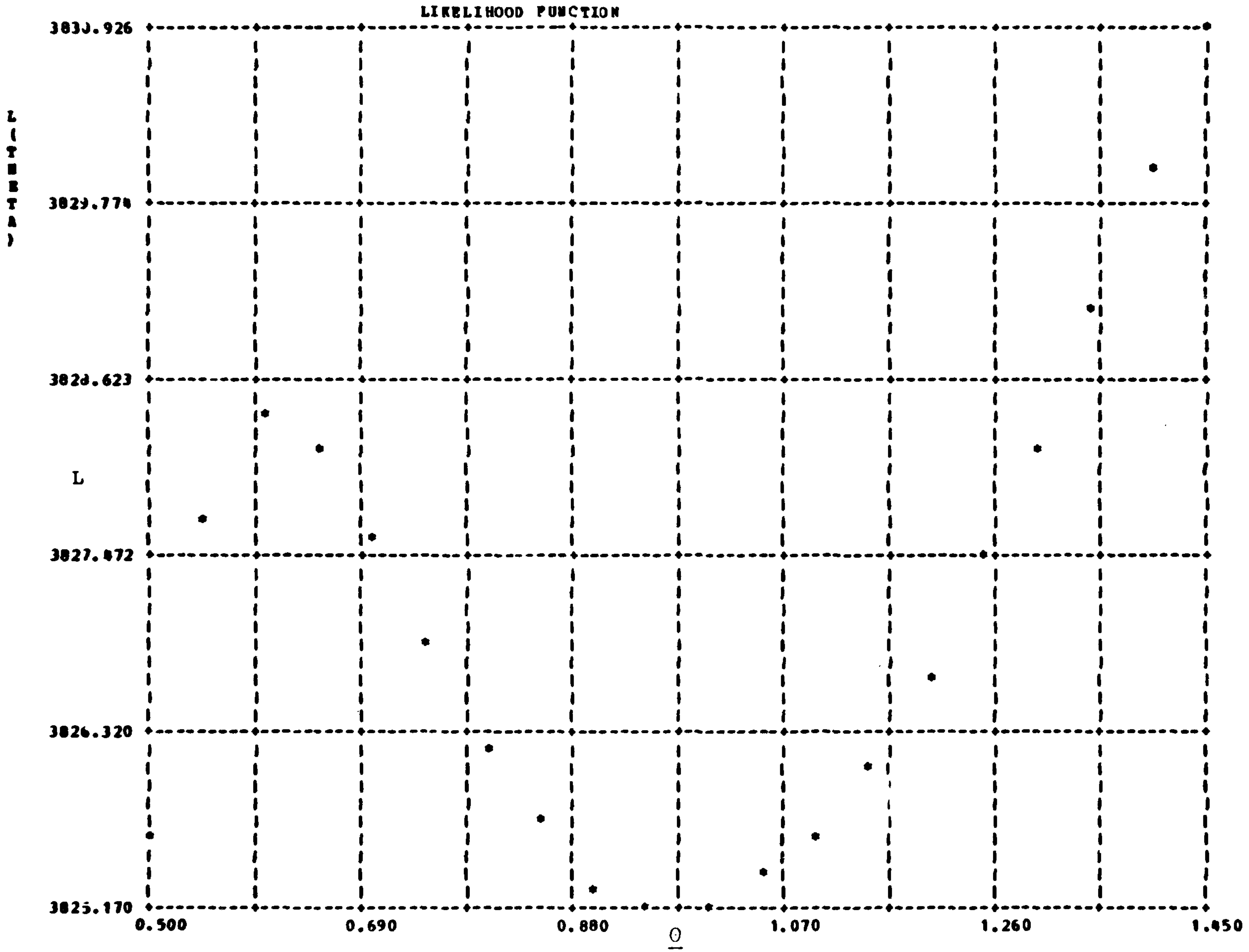

Figure H.10 Plot of the likelihood function for test case 4.

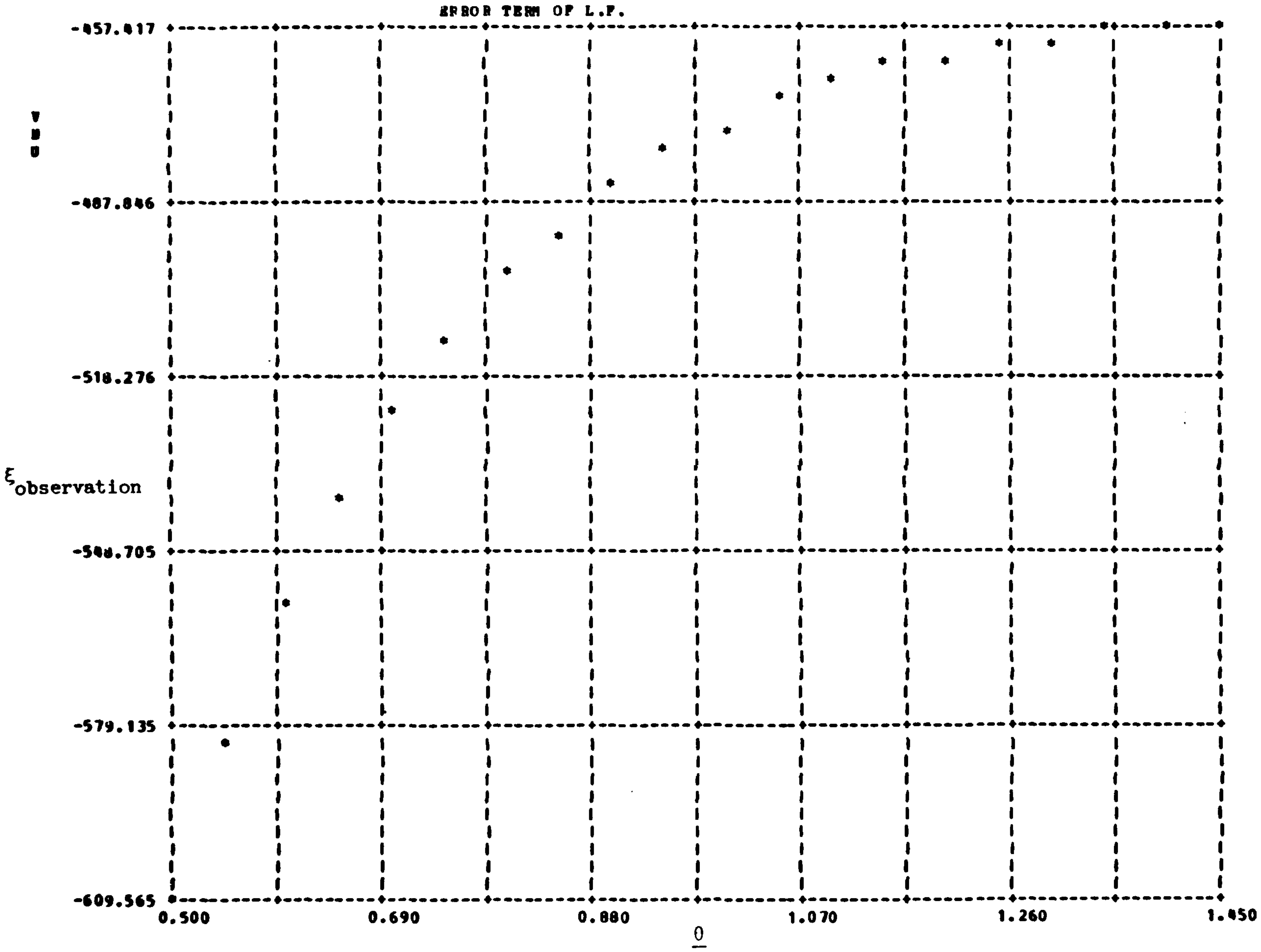

Figure H.11 Plot of the observation term for test case 4.

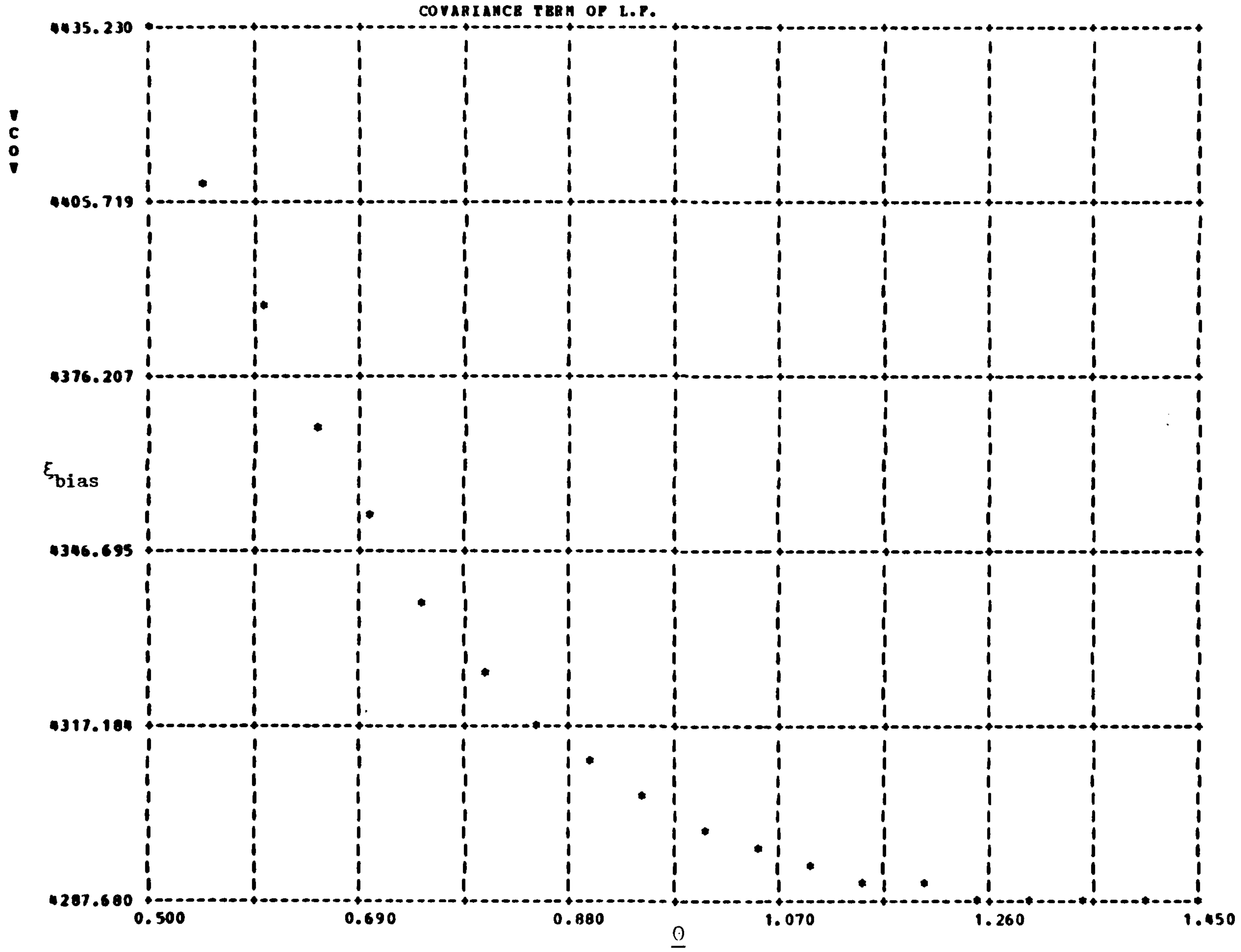

Figure H.12 Plot of the bias term for test case 4.

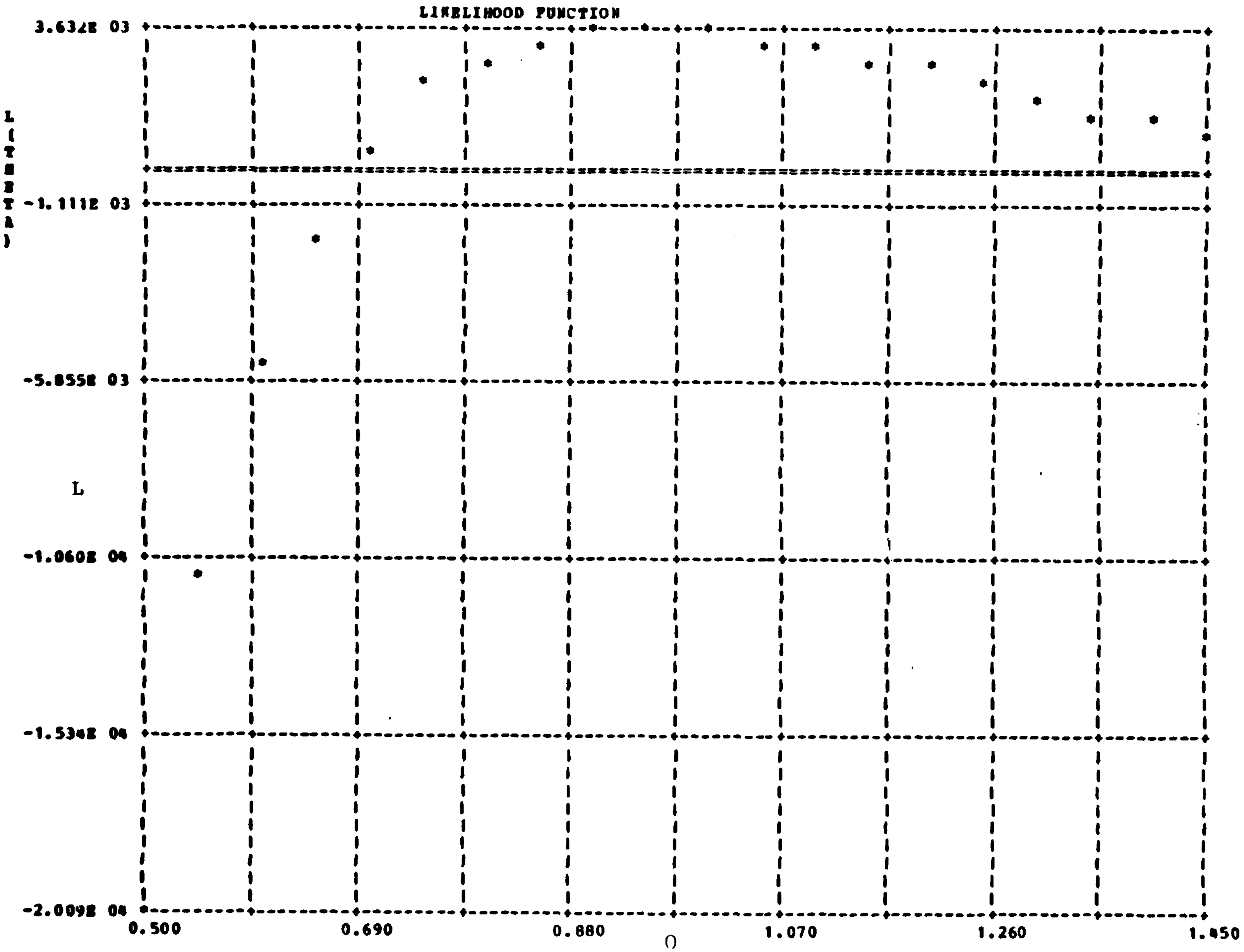

Figure H.13 Plot of the likelihood function for test case 5.

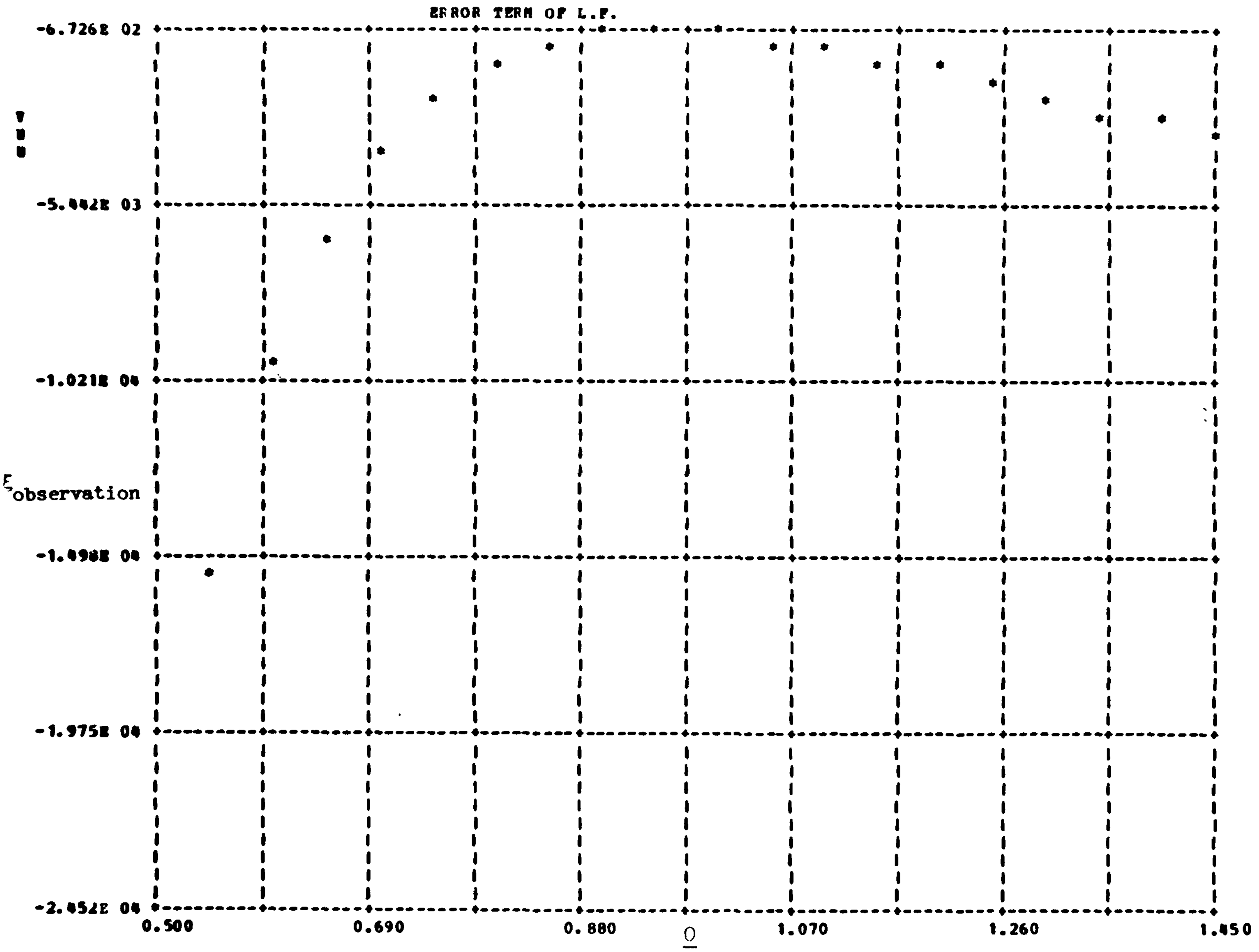

Figure H.14 Plot of the observation term for test case 5.

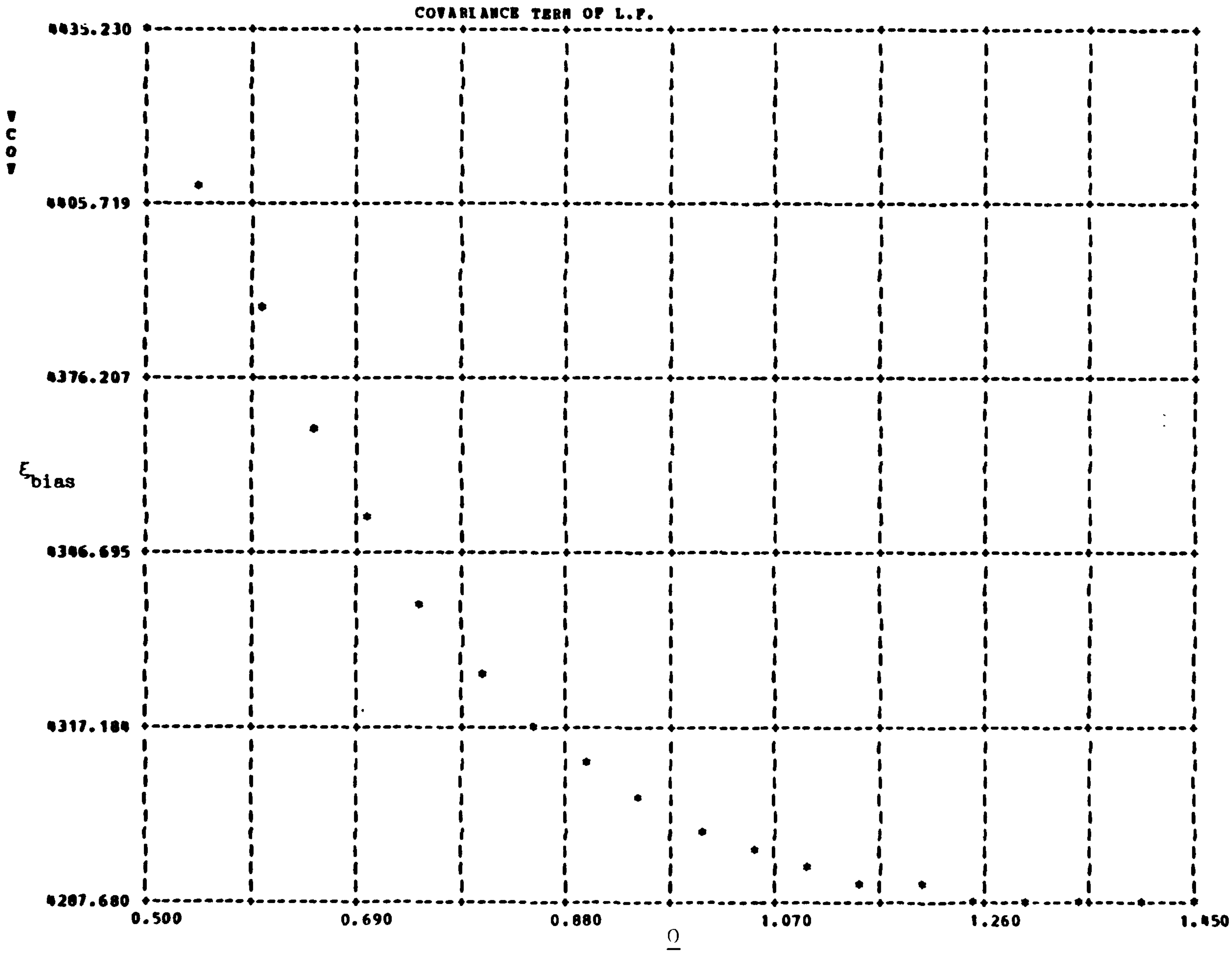

Figure H.15 Plot of the bias term for .est case 5.

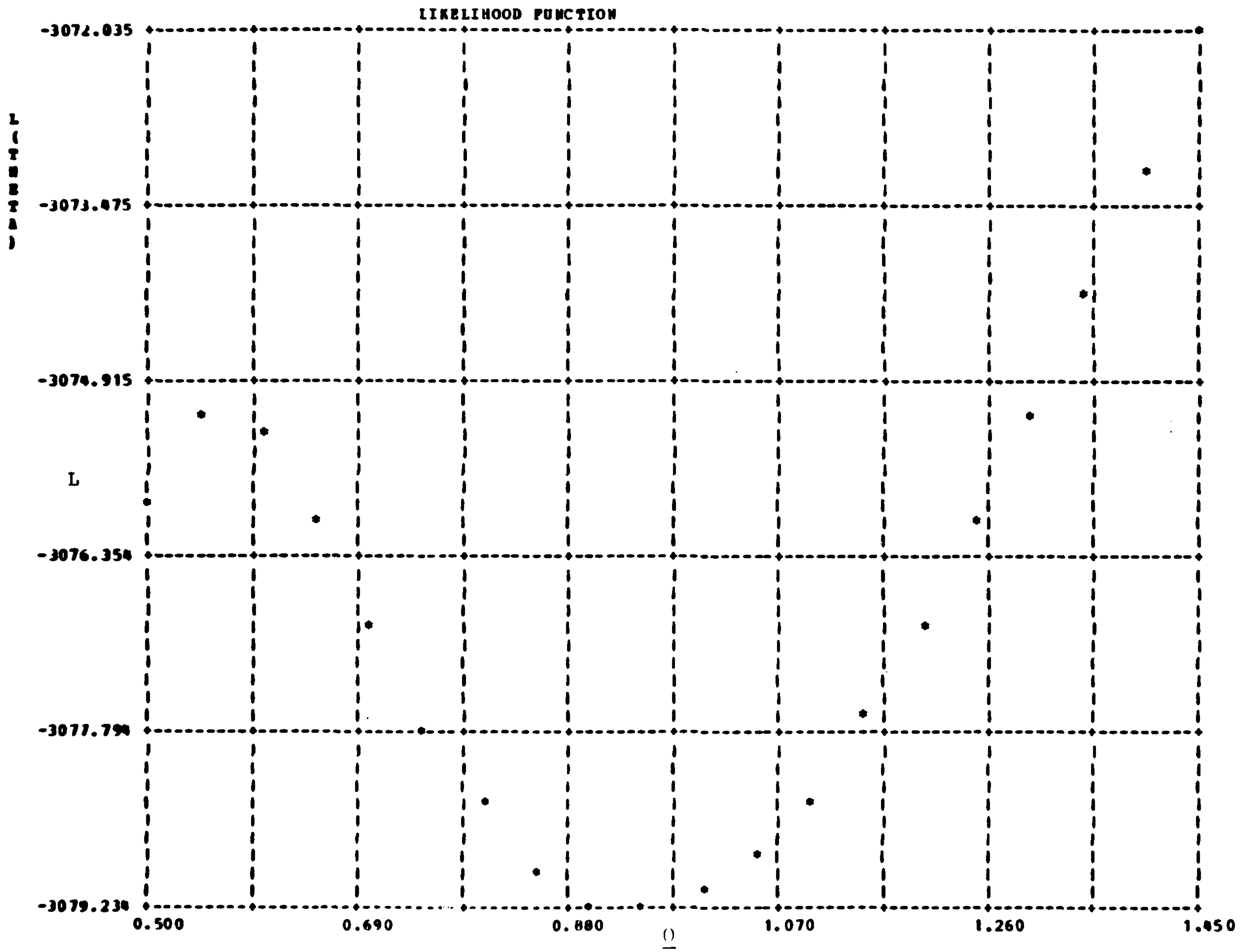

Figure H.16 Plot of the likelihood function for test case 6.

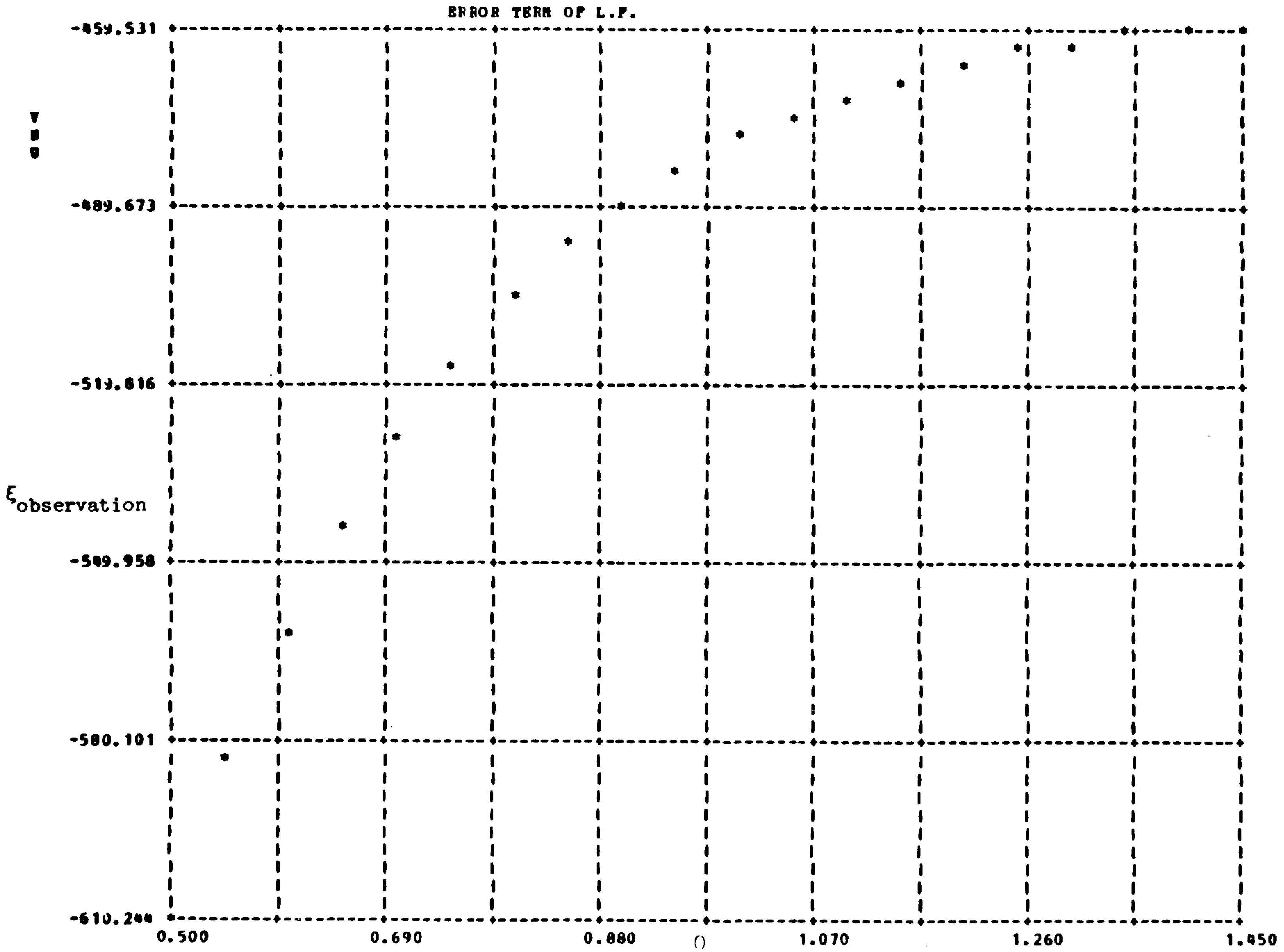

Figure II.17 Plot of the observation term for test case 6.

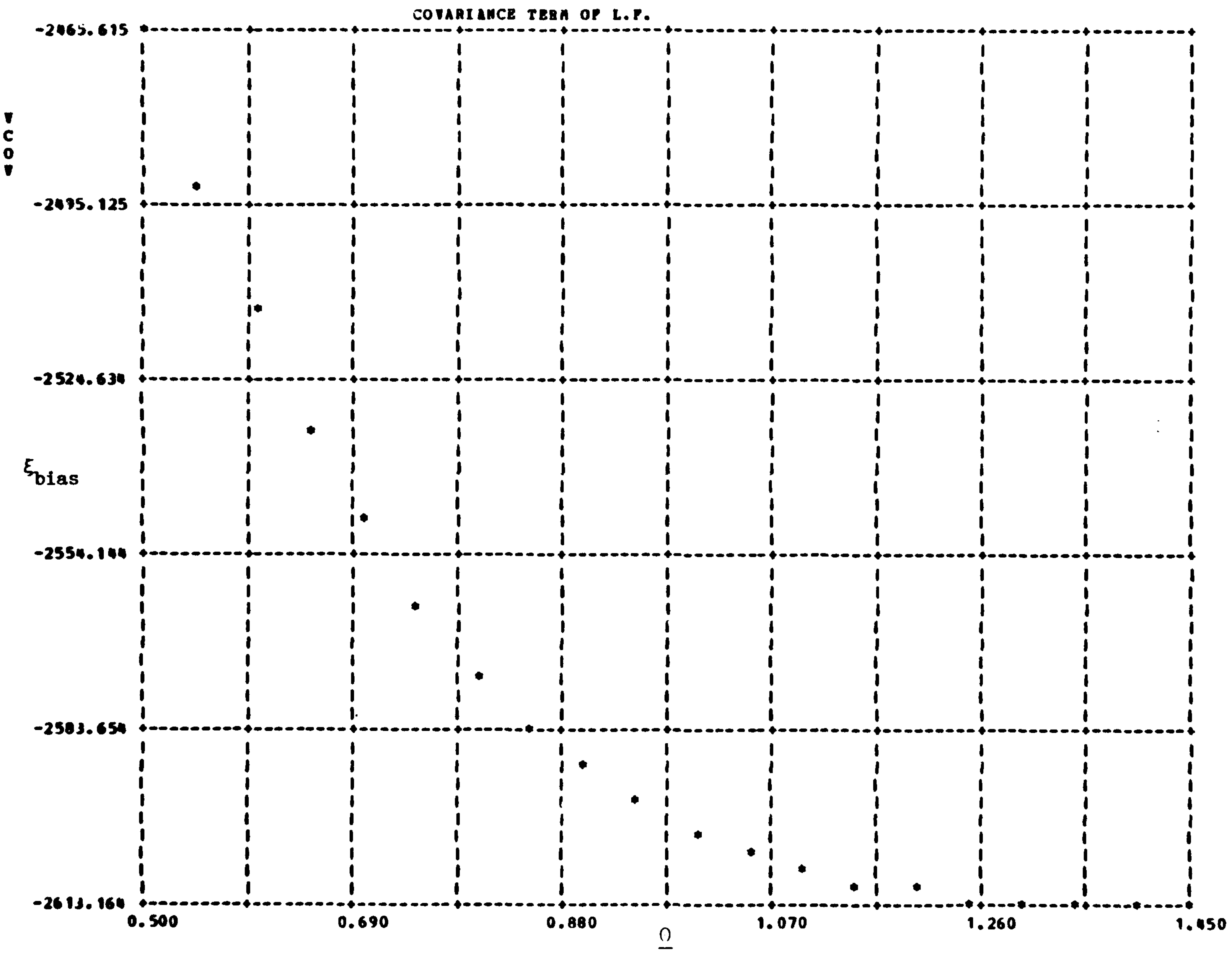

Figure H.18 Plot of the bias term for test case 6.

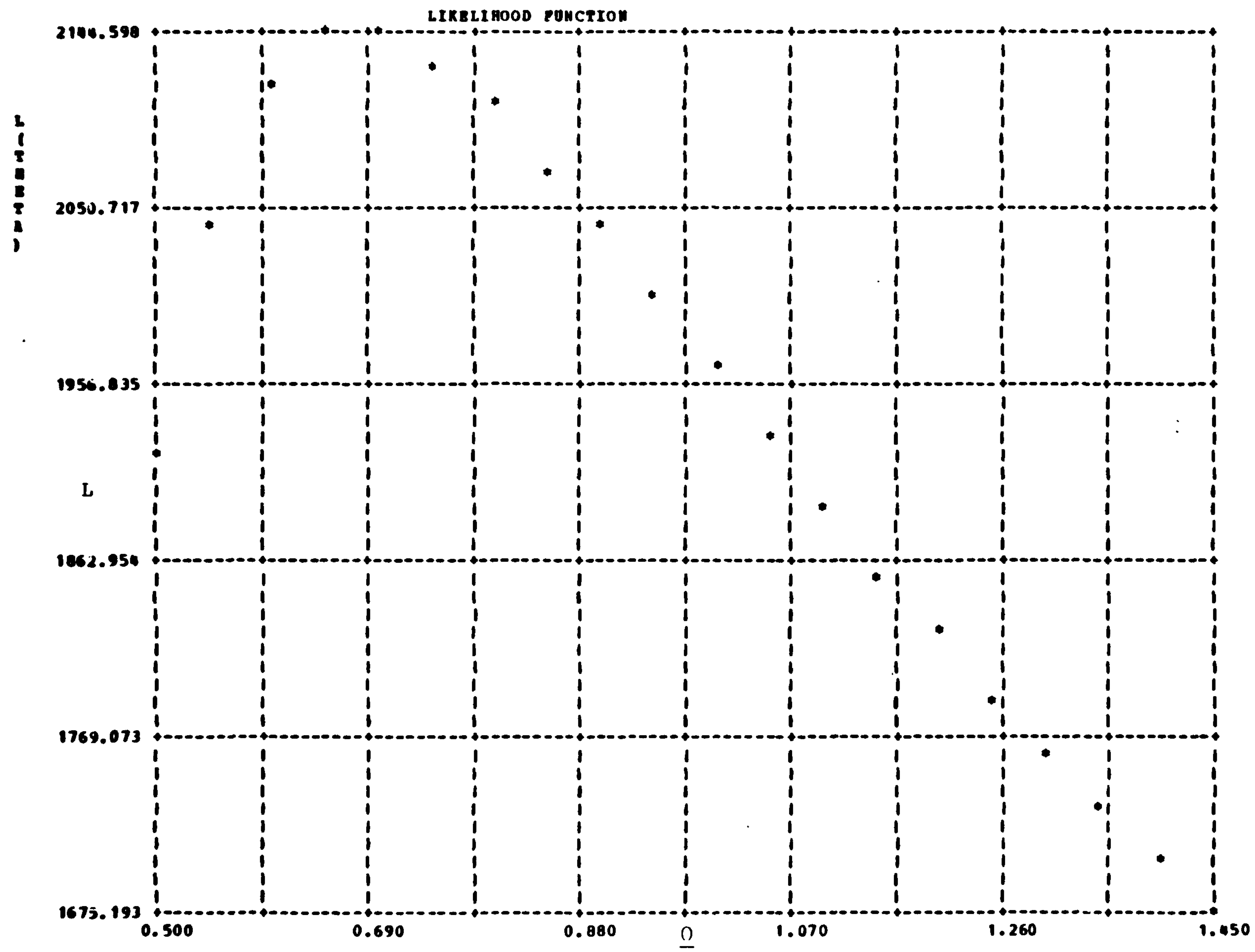

Figure H.19 Plot of the likelih od function for test case 7.

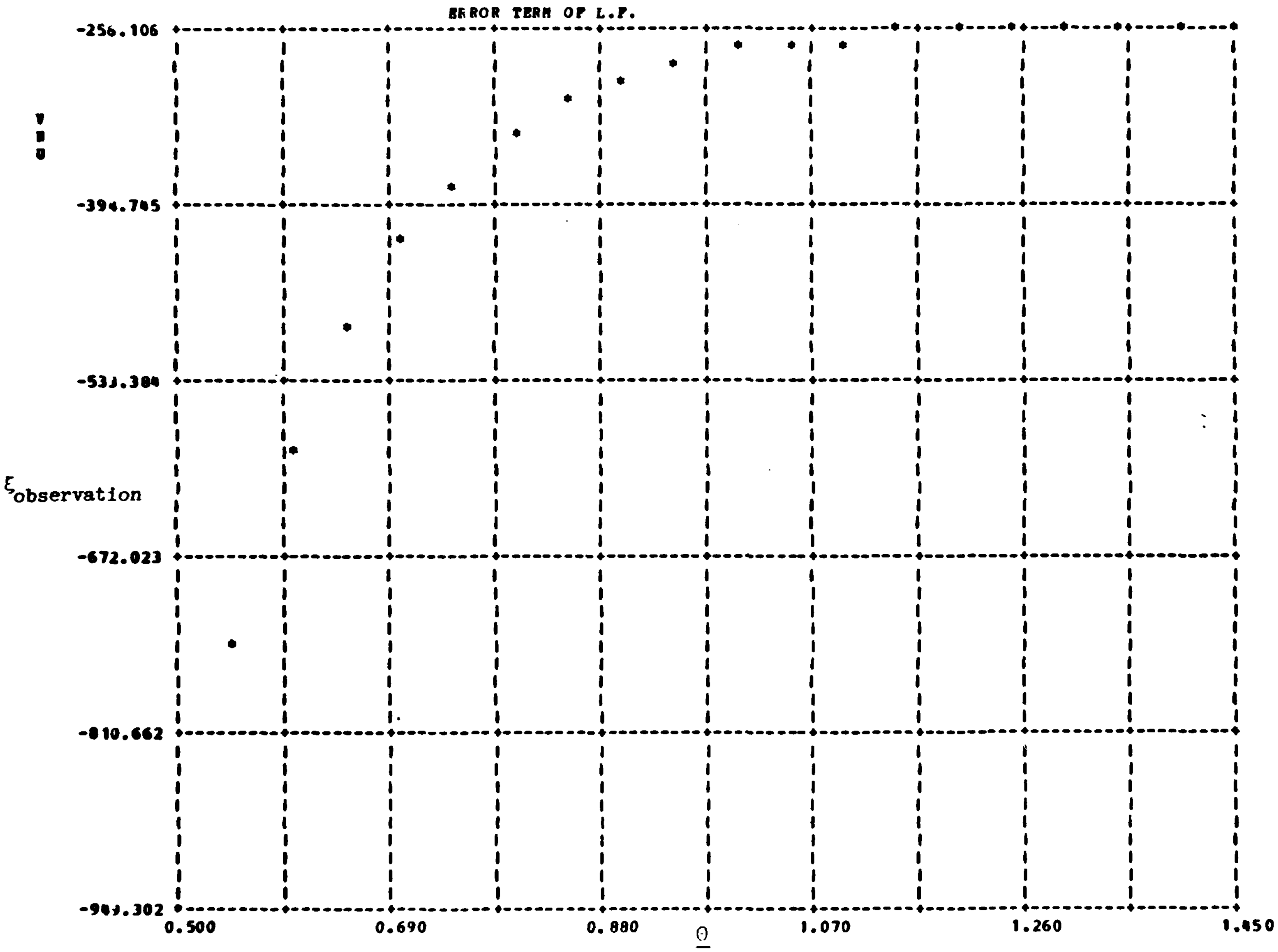

Figure H.20 Plot of the observation term for test case 7.

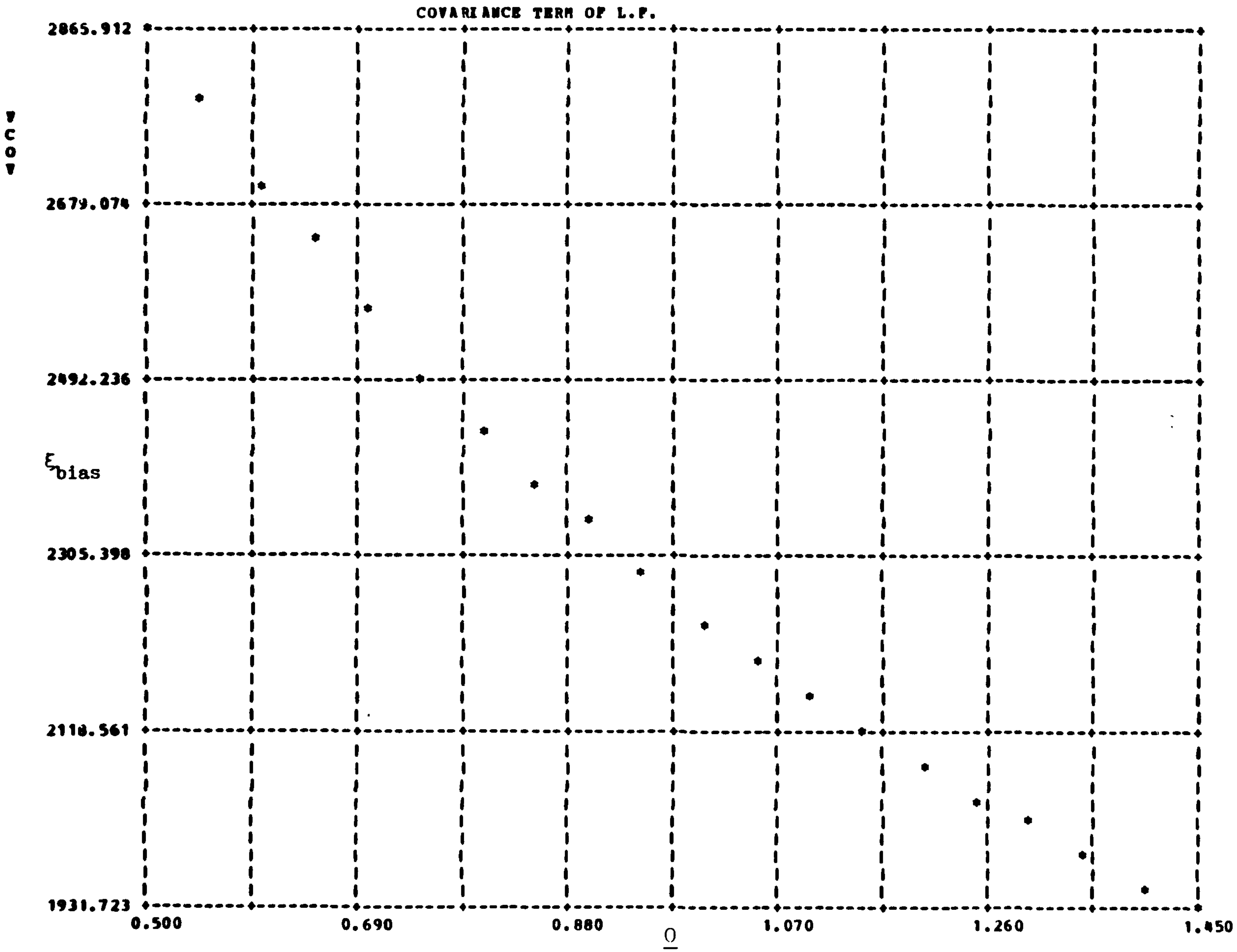

Figure 4.21 Plot of the bias term for test case 7.

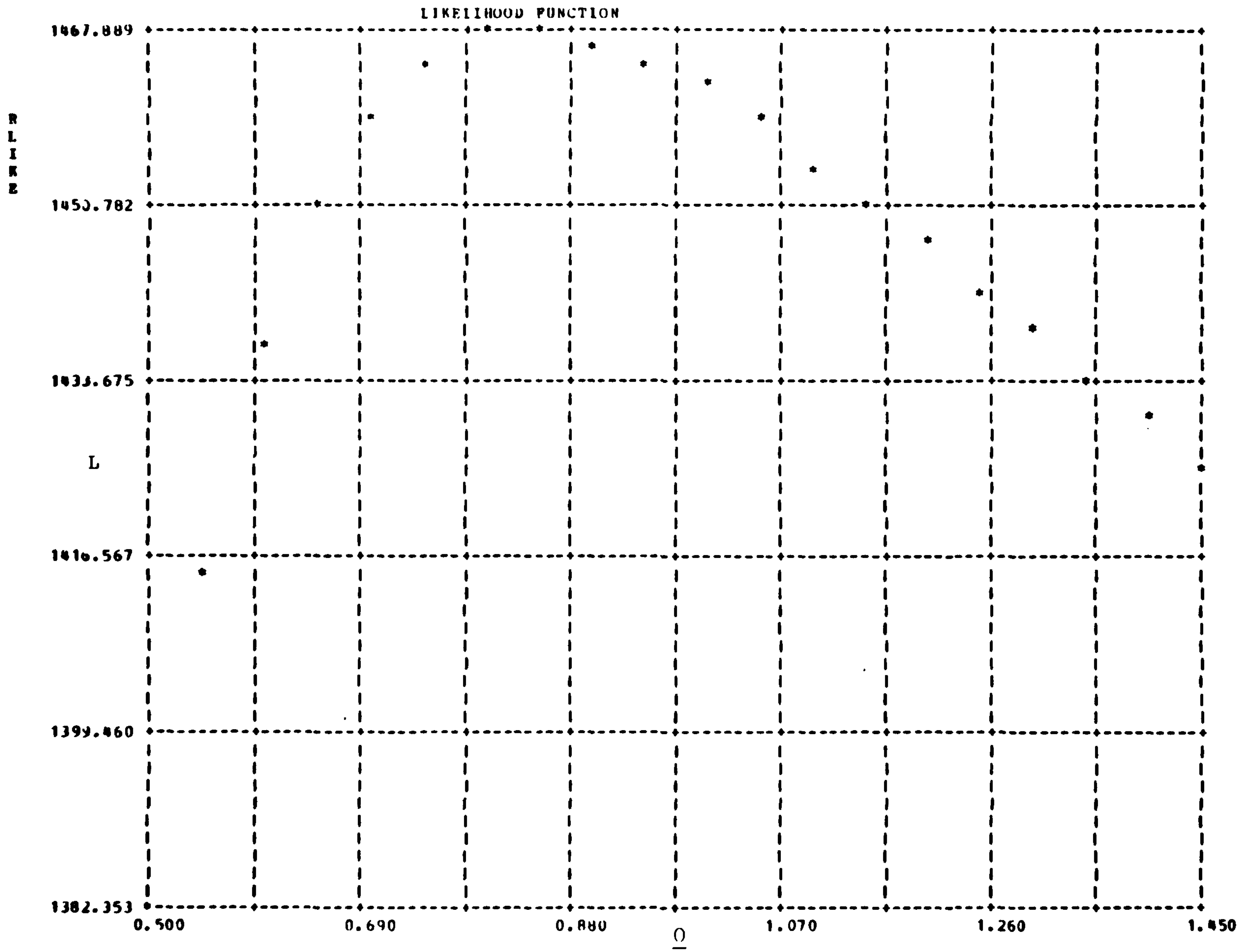

Figure H.22 Plot of the likelihood function for test case 8.

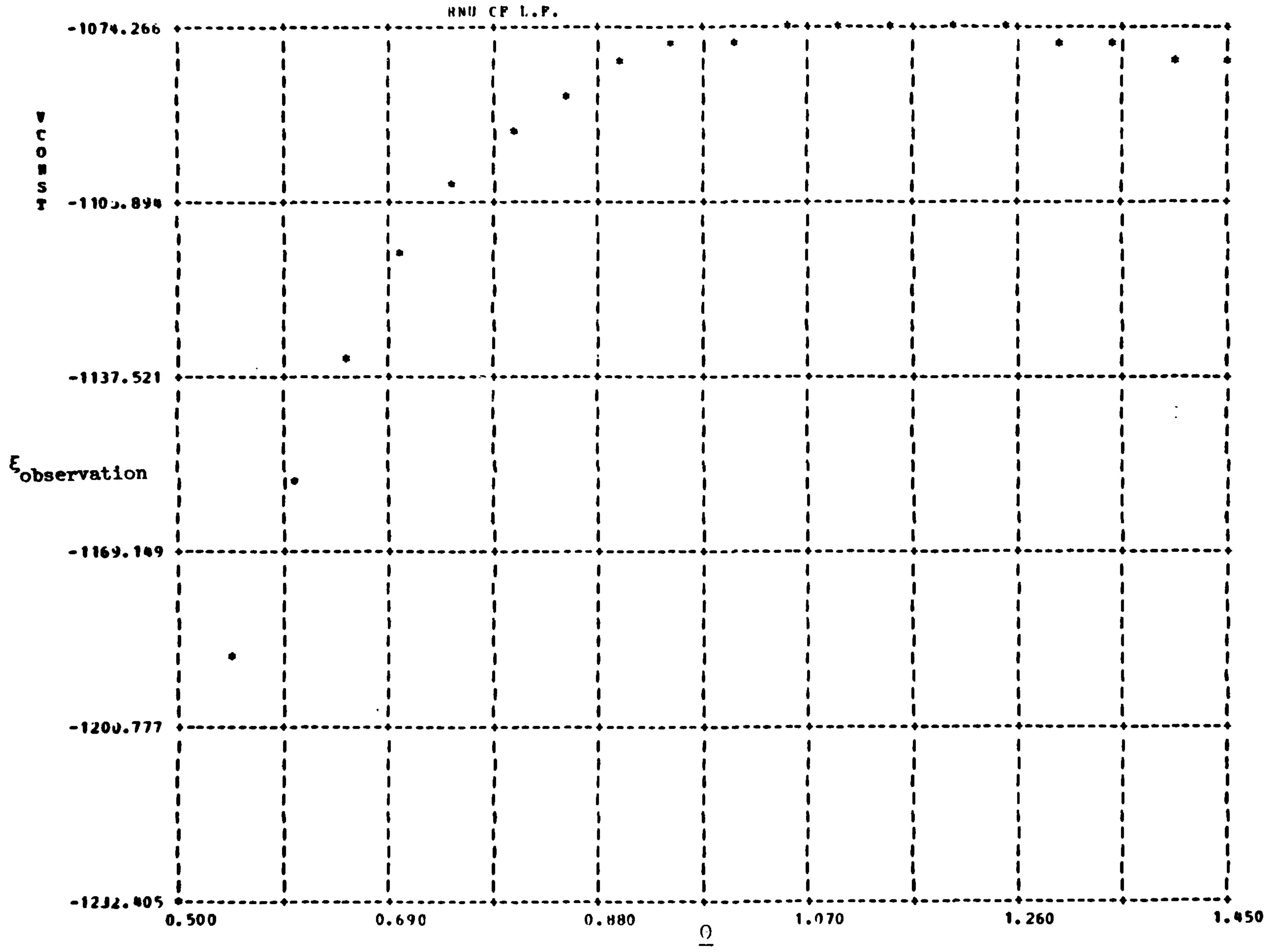

Figure H.23 Plot of the observation term for test case 8.

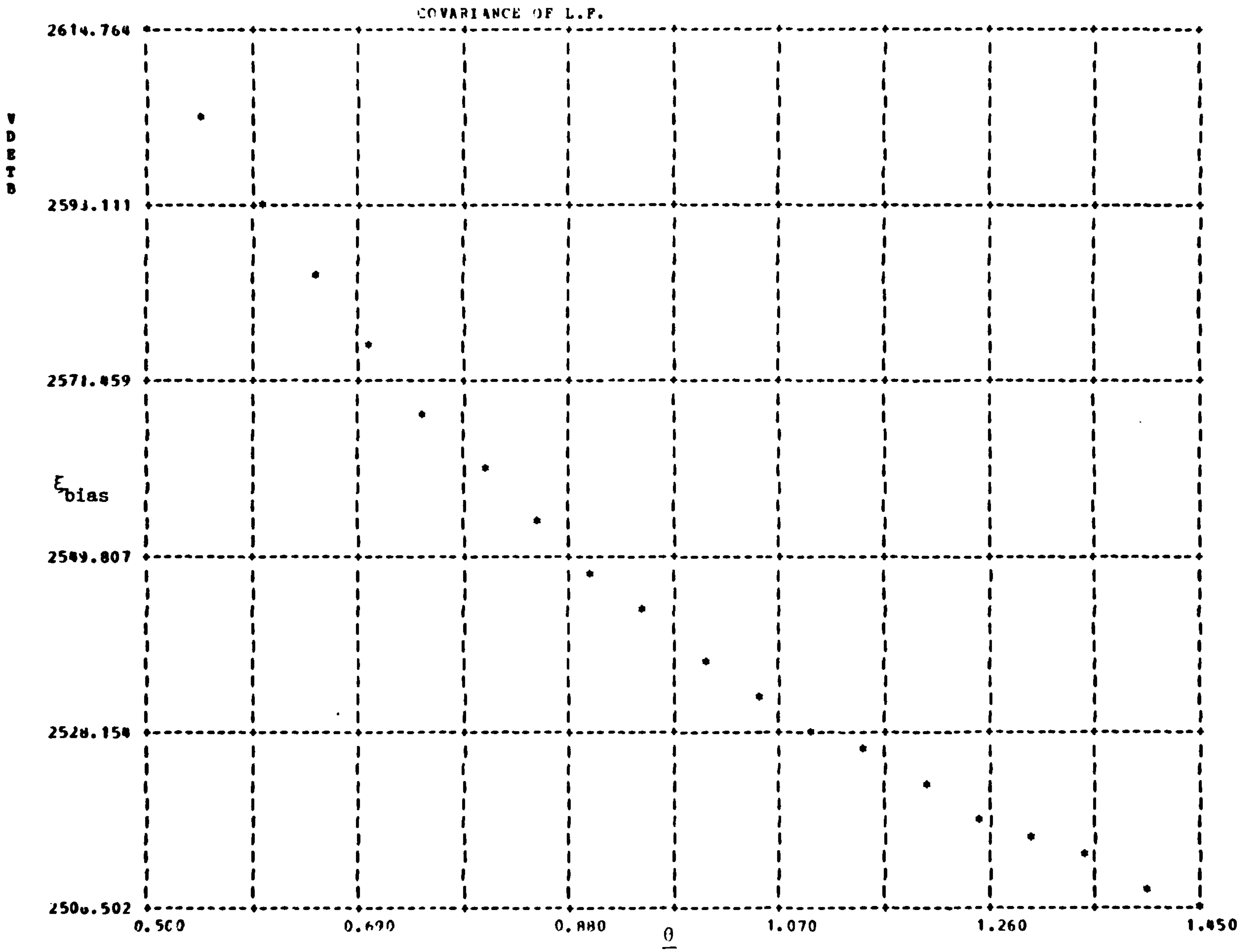

Figure H.24 Plot of the bias term for test case 8.

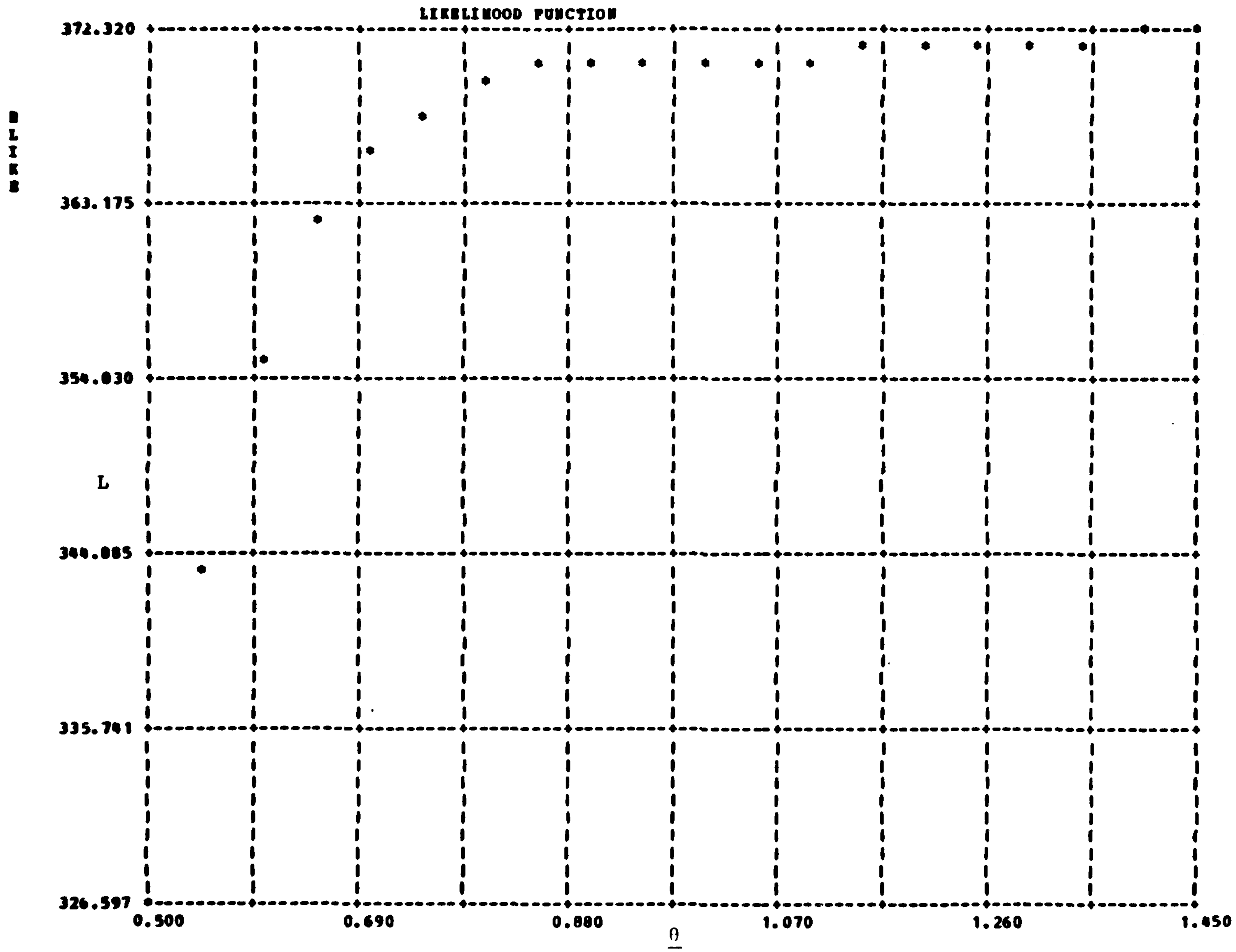

Figure H.25 Plot of the likelihood function for test case 9.

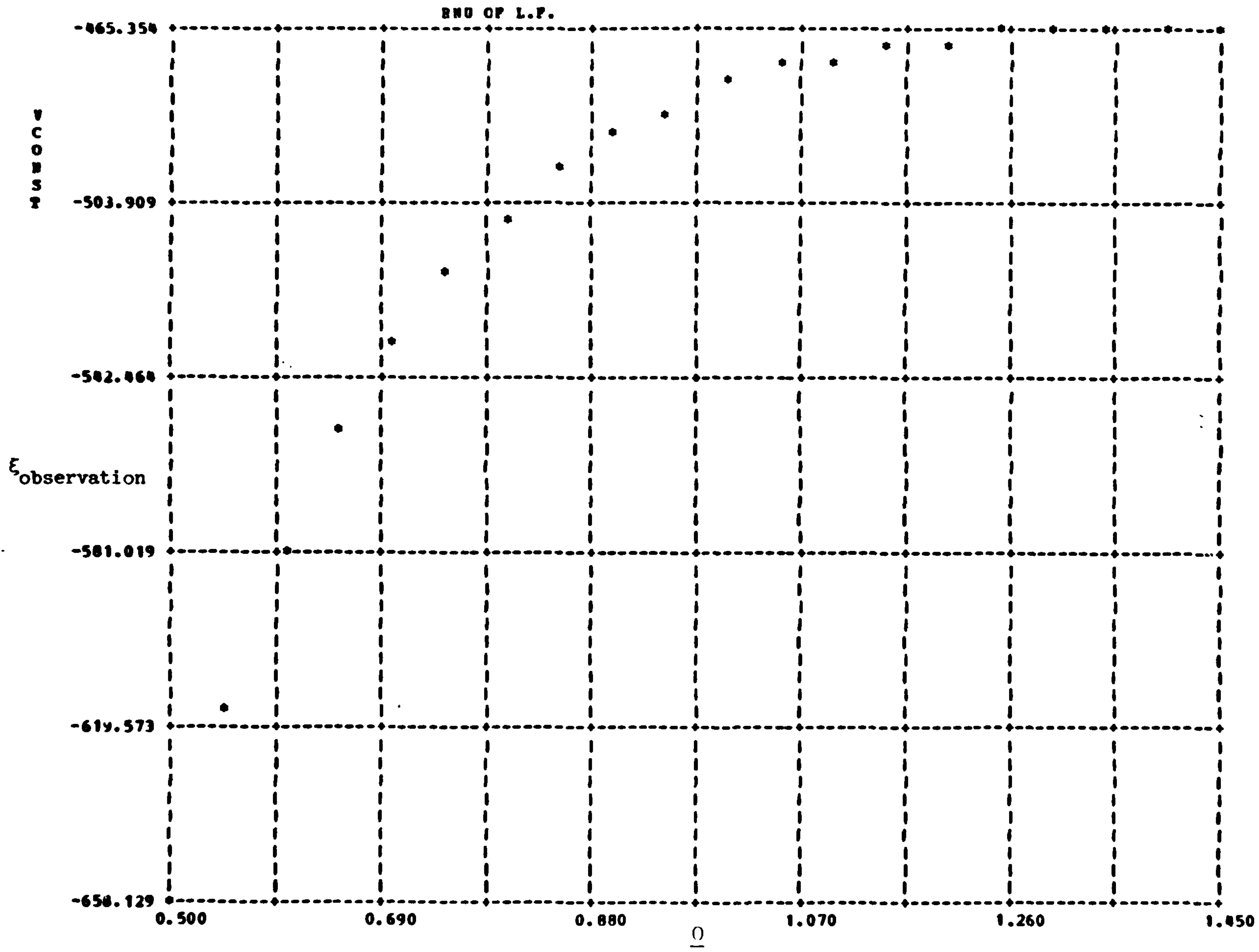

Figure H.26 Plot of the observation term for test case 9.

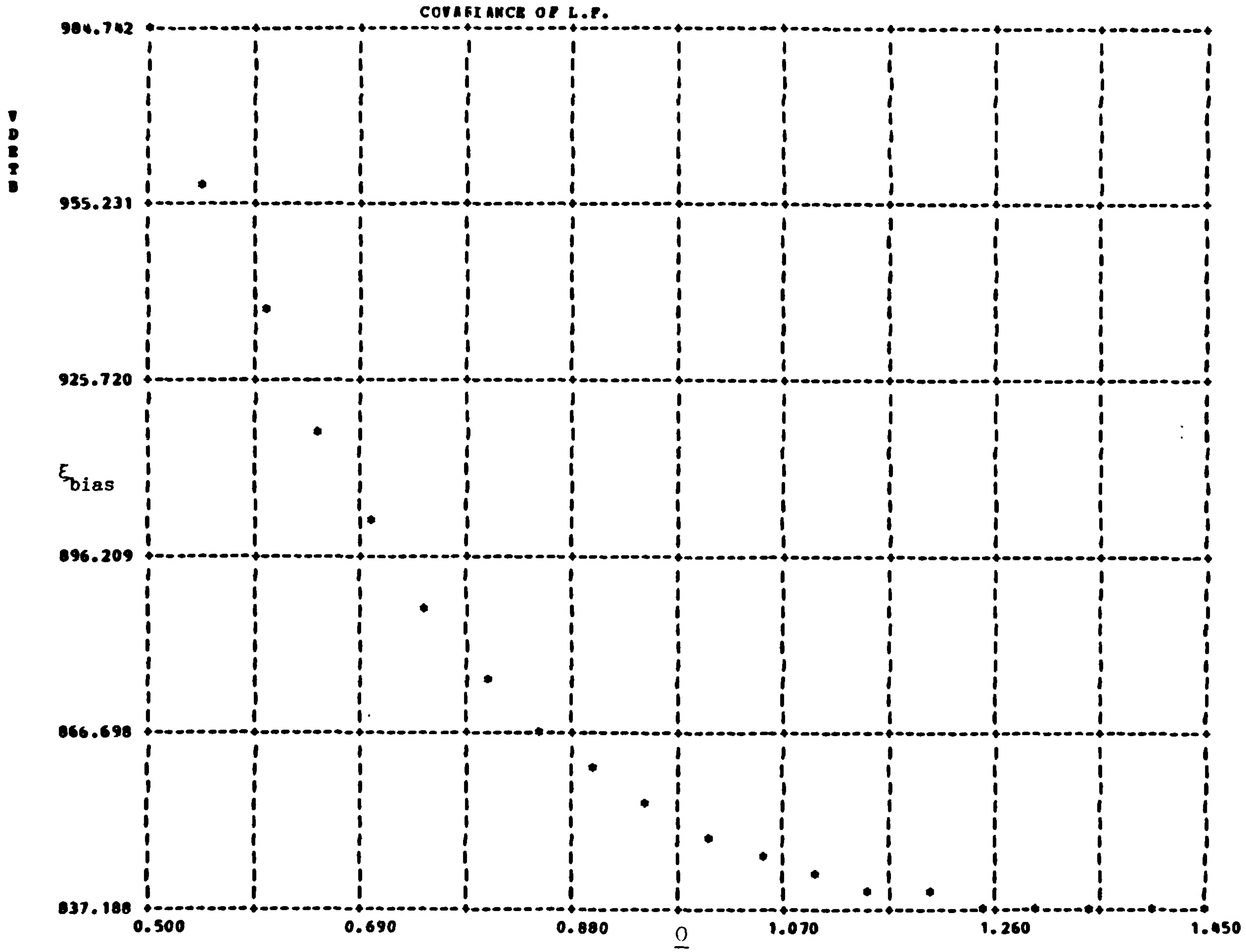

Figure II.27 Plot of the bias term for test case 9.

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